Computational Nuclear Science \& Engineering Course Goal: able to solve problems aided by computer programming

Mathematical
insights into how
algorithms work
Intermediate Level


> Practical programming / debugging skills

Intermediate Level


Model construction \& interpretation of numerical results

Intermediate Level

by Doing Nuclear Science and Engineering Problems!

- incoming MIT NSE graduate students have diverse backgrounds - but, NSE don't need or have time to reinvent the wheels


### 22.107 Course Pre-requisite: $12.010,18.085$

-Assumes basic level of numerical linear algebra, probability theory, finite difference, FFT etc. (if not confident, take 18.085)

- Assumes basic level of programming skills (if not confident, take 12.010)


## Course Approach

- No spoon feeding: lectures provide pointers (references, websites) and examples
- Self study and self-motivated programming a must
- Problem set centric: develop critical analysis and synthetic problem-solving skills by asking them to solve problems with fewer and fewer constraints, end course with completely openended term project
- Arbitrary programming language: ask to show excerpts of source code and intermediate data
- Have fun programming and solving problems.


## Lecture 1: What is Computation?




DNA computer


Quantum computer: D-Wave qubit processor


## Computation is Reverse Mapping From Physical World $\rightarrow$ Mathematics



## Computer Simulation

$$
0,1,+,-, /, \times
$$

## Universal Mathematics



Computer: mapping of abstract, universally applicable mathematics onto evolution of a physical system (internal, external states). This evolution is faster, more controllable (more error free), easier to understand, etc. than unengineered systems.

Pancomputationalism: everything happening in this world is "computation".

Most commercial computers today move electronic charges around.
(However, long-haul communication network moves photons.) specially designed physical system with well-controlled internal and external states and evolutions

Fundamentally, computer network $\equiv$ computer. memory bus on a PC motherboard is a fast network.
Beowulf PC cluster based on ethernet or InfiniBand internal network
Consider computers and the network as a whole: coupled internal states: some strongly coupled, some weakly/intermittently coupled. Analogy between neural network (neuron / synapse) and Internet / cloud computing.

CNSE: the use of computers and networks to facilitate discovery and problem solving in Nuclear Science and Engineering.


From http://www.lanl.gov/history/: The new IBM punched-card machines were devoted to calculations to simulate implosion, and Metropolis and Feynman organized a race between them and the hand-computing group. "We set up a room with girls in it. Each one had a Marchant. But one was the multiplier, and another was the adder, and this one cubed, and all she did was cube this number and send it to the next one," said Feynmann. For one day, the hand computers kept up: "The only difference was that the IBM machines didn't get tired and could work three shifts. But the girls got tired after a while."

Feynmann worked out a technique to run several calculations in parallel on the punched-card machines that reduced the time required. "The problems consisted of a bunch of cards that had to go through a cycle. First add, then multiply, and so it went through the cycle of machines in this room - slowly - as it went around and around. So we figured a way to put a different colored set of cards through a cycle too, but out of phase. We'd do two or three problems at a time," explained Feynman. Three months were required for the first calculation, and Feynman's technique reduced it to two or three weeks.
assembly line in manufacturing $\rightarrow$ instruction pipeline stages in computer architecture

## Clock Cycle



## http://en.wikipedia.org/wiki/Instruction_pipeline



From http://www.computerhistory.org: John von Neumann (left) and Robert Oppenheimer, in front of Princeton's Institute for Advanced Study (IAS) computer. Operational in 1952, the IAS machine was the prototype for the first generation of digital computers.
von Neumann served as consultant in the Manhattan Project.
Neutronics and hydrodynamics are still at the heart of NSE today. So one could say that CNSE was one of the very first applications of modern computing


ENIAC (Electronic Numerical Integrator And Computer, 1946): the first general-purpose Turing-complete electronic computer
at the Moore School of Electrical Engineering, University of Pennsylvania (later transferred to Army's Ballistic Research Laboratory)


First semiconductor transistor (1947, Bell Labs)


John Bardeen


Walter Brattain

Gordon Moore (1965): doubling the density of transistors on integrated circuits every two years


## ARPANET/Internet ( $\rightarrow$ TCP/IP): late 1960s



## World Wide Web (Tim Berners-Lee, 1991, CERN) <br> $\rightarrow$ HyperText Markup Language (HTML) $\uparrow$




Welcome to NCSA Mosaic, an Internet information bro Mosaic was developed at the National Center for Sup University of Illinais in --> Urbana-Champaign. NCS The Board of Trustees of the University of Illinais UI.

Vannevar Bush 1945 essay, "As We May Think." a theoretical machine called "memex," to enhance human memory by allowing the user to store and retrieve documents linked by associations.


## Scientific Inquiry

## Experimentation

replication of the physical system of interest to repeat its evolution, e.g. replicate a much smaller, but otherwise very similar, piece of the real world to simulate the real world
$3^{\text {rd }}$ pillar:
Computation
"mapping" is onto neither the human brain, nor a smaller replication of physical system, but in silico - a wellcontrolled physical system (electronic computer) with no external resemblance to the physical system of interest.

## Theory

reduction of natural processes to humancomprehensible logic, and then aided by simple calculations, to predict natural processes
"in-brain mapping"

## Well-controlledness of today's digital computers is outstanding:

Almost all physical experiments we do subjected to noise. But impression of digital computation: no noise.

Thermal fluctuations (true randomness) are entirely filtered out, by design of electronic circuits. electronics in outer space an exception

Pseudo randomness needs to be artificially introduced when needed in simulations.

## Advantages of perform mapping in silico

- Compared to in-brain mapping: vast advantages in speed, accuracy, data storage, ...
- Compared to physical world mapping: cost, better control of initial and boundary conditions (parametric studies), rich data (access to all internal states), ...


## Disadvantages

- Retains only key pieces of the physics in mapping - loss of physics: no material deformation when modeling thermal conduction BTW, this is the same for in-brain mapping.

Double-edged sword: This loss-of-physics disadvantage is also tied to the advantage of "better control of initial and boundary conditions": when modeling surface chemical reactions under ultra-high vacuum conditions, not worry about vacuum leaks like experimentalists must

- "Curse of dimensionality"

Many real-world processes are still too complex to be simulated in silico, at a level we would like to simulate them.

## World of Atoms \& Electrons

$$
\begin{aligned}
& i \hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t)=-\frac{\hbar^{2}}{2 m} \nabla^{2} \Psi(\mathbf{r}, t)+V(\mathbf{r}) \Psi(\mathbf{r}, t) \\
& \quad\left(\beta m c^{2}+\sum_{k=1}^{3} \alpha_{k} p_{k} c\right) \psi(\mathbf{x}, t)=i \hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}
\end{aligned}
$$



Nobel Prize in Physics 1933

$$
\begin{aligned}
& + \\
& \text { http://top500.org/ }
\end{aligned}
$$

First-Principles Calculations Atomistic Simulations: "Game of God"

Materials Chemistry Life Energy


## Discipline Electronic structure


http://tu-freiberg.de/fakult4/imfd/cms/Multiscale/multiscale.html

## Multiscale Modeling Framework



Multiscale modeling framework for catalytic processes that exhibit strong coupling between scales. Adapted from Vlachos, Adv. Chem. Eng. 30, 1-61 (2005)

## To survive as modeler: first, be humble

- Must respect experimental data

Even if you do not do the experiment, try best to understand

- how the experiment was actually done
- what were the raw data
- confidence level about data
- respect raw data, not necessarily experimentalist's interpretation
- Must respect theory

Without theory, computation is blind

- Best approach to do science and engineering is symbiosis of all three "mappings". Experiments are the ultimate check; human-comprehensible form is the ultimate desirable form; but computers can help get us there!

"Four color map" theorem first proven using computer (1976). Logic Power
The computer proof spreads over 400 pages of microfiche.
"a good mathematical proof is like a poem - this is a telephone directory!" Appel and Haken (UIUC) agreed the proof was not "elegant, concise and completely comprehensible by a human mathematical mind".


## 777 Freighter

Boeing 777: the first commercial aircraft to be designed entirely on computer (CAD CATIA, 1994): Number-crunching power


IBM Deep Blue beat chess world champion Garry Kasparov (1997) Logic / Computing power

WATSON vs. HUMANS

| Round | Watson |  | Rutter | Jennings |
| :---: | :---: | :---: | :---: | :---: |
| 1 (mons) | $\$ 5000$ | $\$ 5000$ | $\$ 200$ |  |
| $2($ (Tues) | $\$ 35,734$ | $\$ 10,800$ | $\$ 4,800$ |  |
| 3 (Wed) | $\$ 77,147$ | $\$ 21,600$ | $\$ 24,000$ |  |
| Final prize | $\mathbf{\$ 1 , 0 0 0 , 0 0 0}$ | $\$ 200,000$ | $\$ 300,000$ |  |



IBM Watson won quiz show Jeopardy! (2011)

Data Power

Natural
Language Processing


## DUE TO ENORMOUS ENERGY CONSUMPTION, DATA CENTERS' GARBON FOOTPRINT IS ALSO SURPRISINGLY HICH AND GROWING

## Key points on data centers' greenhouse gas emissions

- Data center electricity consumption is almost $.5 \%$ of world production*
- Average data center consumes energy equivalent to $\mathbf{2 5 , 0 0 0}$ households
- Worldwide energy consumption of DC doubled between 2000 and 2006
- Incremental US demand for data center energy between now and 2010 is equivalent of 10 new power plants
- $90 \%$ of companies running large data centers need to build more power and cooling in the next 30 months

- Including custom-designed servers (e.g., Google, Yahoo)

Source: Financial Times; Gartner report 2007; Stanford University; AMD; Uptime Institute; McKinsey analysis
"Worldwide, the digital warehouses use about 30 billion watts of electricity, roughly equivalent to the output of 30 nuclear power plants", Power, Pollution and the Internet - by James Glanz, New York Times, September 22, 2012

Hot theoretical problem: Computation and the $2^{\text {nd }}$ law of Thermodynamics
...and a day may come when NSE need to show up to save computation

## Bibliometric approach (http://apps.webofknowledpe.com.libproxv.mit.edu) in

 Nuclear Science and Engineering- Nuclear Science and Engineering (ANS, 1956)
- Nuclear Engineering and Design (Elsevier)
- Nuclear Technology (ANS)
- Progress in Nuclear Energy (Elsevier)
- Annals of Nuclear Energy (Elsevier)
- Nuclear Fusion (IAEA / Institute of Physics-UK)
- Physics of Plasmas (American Institute of Physics)
- Journal of Nuclear Materials (Elsevier)
- Health Physics (Health Physics Society)
- Nuclear Instruments and Methods in Physics Research


## Broader Appeal Journals

- Physical Review Letters (American Physical Society)
- PNAS (National Academy of Sciences)
- Science (American Association for the Advancement of


## Science)

- Nature (Macmillan-UK)

Comparing journals (http://admin-apps.webofknowledge.com/JCR/JCR)

|  |  |  |  | JCR Data i) |  |  |  |  |  | Eigenfactor ${ }^{\circledR}$ Metrics $\mathbf{i}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mark | Rank | Title <br> (linked to journal information) | ISSN | Total Cites | Impact Factor | 5-Year <br> Impact <br> Factor | Immediacy Index | Articles | Cited Halflife | $\begin{aligned} & \text { Eigenfactor }{ }^{\circledR} \\ & \text { Score } \end{aligned}$ | $\begin{gathered} \text { Article } \\ \text { Influence }{ }^{\circledR} \\ \text { Score } \end{gathered}$ |
| $\square$ | 1 | NATURE | $\begin{aligned} & 0028- \\ & 0836 \end{aligned}$ | 526505 | 36.280 | 36.235 | 9.690 | 841 | 9.4 | 1.65524 | 20.373 |
| $\square$ | 1 | J NUCL MATER | $\begin{aligned} & 0022- \\ & 3115 \end{aligned}$ | 16255 | 2.052 | 2.060 | 0.298 | 1053 | 7.9 | 0.03217 | 0.587 |



## 2008 impact factor of journal $\mathrm{X} \equiv$

 \# citations in year 2008 to articles published in 2006 and 2007 in journal X / \# articles published in 2006 and 2007 in journal X"Impact factor $=2.1 "$ roughly means 2.1 citations/year in 1.5 years after publication

| Field (Size) | $\overline{\mathrm{IF}}$ |  | Field (Size) | $\overline{\mathrm{IF}}$ |
| :--- | :--- | :--- | :--- | :--- |
| Molecular and Cell Biology (511) | 4.763 |  | Dentistry (43) | 1.284 |
| Astronomy and Astrophysics (25) | 4.295 |  | Orthopedics (72) | 1.226 |
| Gastroenterology (40) | 3.475 |  | Telecommunication (37) | 1.192 |
| Rheumatology (20) | 3.348 |  | Applied Acoustics (36) | 1.171 |
| Neuroscience (224) | 3.252 |  | Crop Science (61) | 1.04 |
| Medicine (766) | 2.896 |  | Business and Marketing (101) | 1.035 |
| Chemistry (145) | 2.61 |  | Geography (56) | 0.986 |
| Pharmacology (28) | 2.331 |  | Information Science (23) | 0.918 |
| Psychiatry (178) | 2.294 |  | Agriculture (56) | 0.882 |
| Urology (23) | 2.132 |  | Anthropology (62) | 0.872 |
| Medical Imaging (84) | 2.043 |  | Material Engineering (107) | 0.826 |
| Pathology (28) | 1.991 |  | Economics (159) | 0.823 |
| Physics (503) | 1.912 |  | Fluid Mechanics (107) | 0.804 |
| Ophthalmology (36) | 1.905 |  | Probability and Statistics (57) | 0.796 |
| Environmental Health (73) | 1.871 |  | Veterinary (77) | 0.767 |
| Analytic Chemistry (129) | 1.789 |  | Sociology (96) | 0.715 |
| Geosciences (224) | 1.768 |  | Media and Communication (24) | 0.69 |
| Law (71) | 1.657 |  | Control Theory (64) | 0.681 |
| Ecology and Evolution (349) | 1.555 |  | Political Science (99) | 0.68 |
| Parasitology (38) | 1.527 |  | Computer Science (124) | 0.631 |
| Environmental Chemistry and Microbiology (181) | 1.505 | Education (86) | 0.59 |  |
| Computer Imaging (31) | 1.446 | Mathematics (149) | 0.556 |  |
| Dermatology (38) | 1.427 |  | Operations Research (62) | 0.542 |
| Psychology (210) | 1.387 | History and Philosophy of Science (32) | 0.456 |  |
| Chemical Engineering (75) | 1.29 | History (23) | 0.416 |  |

## Impact factor varies A LOT across fields, even sub-fields. One has to be very careful in using citation statistics when comparing journal/researcher across different fields.

# LETTERS 

## Data, Web and Informatics

## Detecting influenza epidemics using search engine query data <br> ${ }^{1}$ 'Google Inc., 1600 Amphitheatre Parkway, Mountain View, California 94043, USA. <br> ${ }^{2}$ Centers for Disease Control and Prevention, 1600 Clifton Road, NE, Atlanta, Georgia 30333, USA.

Jeremy Ginsberg ${ }^{1}$, Matthew H. Mohebbi ${ }^{1}$, Rajan S. Patel ${ }^{1}$, Lynnette Brammer ${ }^{2}$, Mark S. Smolinski ${ }^{1}$ \& Larry Brilliant ${ }^{1}$


Data available as of 12 May 2008


ILI: influenza-like illness physician visits
45 Google queries (informatics, no actual medicinal knowledge), like "robitussin", "symptoms", "fever", "pnumonia", "amoxicillin", "strep throat", ...

Google estimates were consistently 1-2 weeks ahead of CDC ILI surveillance reports.

## HyperText Markup Language (HTML)

demo.html
<HTML>
<HEAD><TITLE>CNSE test</TITLE></HEAD>
<FONT SIZE=+2>Instructor <a href=http://li.mit.edu>Ju Li</A></FONT>
<P>
You can find menp introduction <a href=http://mcnp.lanl.gov/>here</a>.
<p>A cool figure:
<br><a href=Illustration2.jpg><img src=Illustration2.jpg width=150></a><P>Byebye! </HTML>
<!-- http://li.mit.edu/S/CNSE/demo.html -->

host name
(DNS $\rightarrow$ 18.54.1.57)
Press Ctrl+U (Cmd-Opt-U on mac) to view html source


## $k_{\text {in }}($ a.html $)=2$

## $k_{\text {out }}($ a.html $)=3$

PageRank (U.S. Patent 6,285,999), filed by Larry Page and Sergey Brin at Stanford University in 1996, propelled Google to a Mkt Cap 0.2 Trillion dollars company in 2012.

In essence, web pages are ranked by their $k_{\text {in }}$

## Lecture 2: Nature of the Network



Figure 1 Distribution of links on the World-Wide Web. a, Outgoing links (URLs found on an HTML document); $\mathbf{b}$, incoming links (URLs pointing to a certain HTML document). Data were obtained from the complete map of the nd.edu domain, which contains 325,729 documents and 1,469,680 links. Dotted lines represent analytical fits used as input distributions in constructing the topological model of the web; the tail of the distributions follows $P(k) \approx k^{-\gamma}$, with $\gamma_{\text {out }}=2.45$ and $\gamma_{\mathrm{n}}=2.1$. c, Average of the shortest path between two documents as a function of system size, as predicted by the model. To check the validity of our predictions, we determined $d$ for documents in the domain nd.edu. The measured $\left\langle d_{\text {nd.edu }}\right\rangle=11.2$ agrees well with the prediction $\left\langle d_{3 \times 10^{5}}\right\rangle=11.6$ obtained from our model. To show that the power-law tail of $P(k)$ is a universal feature of the web, the inset shows $P_{\text {out }}(k)$ obtained by starting from whitehouse.gov (squares), yahoo.com (triangles) and snu.ac.kr (inverted triangles). The slope of the dashed line is $\gamma_{\text {out }}=2.45$, as obtained from nd.edu in $\mathbf{a}$.

Albert, Jeong, Barabasi, "Internet - Diameter of the World-Wide Web," Nature 401 (1999) 130. \% 1382 cites

In this lecture, show connections between:

1. Power-law distribution ("scale-free" behavior)
2. "Network science" model: growth and preferential attachment
3. Often, hierarchical organization of society and nature


| 1-Bill Gates (\$ 59 Billion) - Microsoft | 5 | 59.0 |
| :---: | :---: | :---: |
| 3-Larry Ellison (\$ 33 Billion) -Orade | 5 | 33.0 |
| 13-Jeff Bezos (\$ 19.1 Billion) - Amazoncom | 5 | 19.1 |
| 14-Mark Zuckerberg (\$ 17.5 Billion) - Facebook | 5 | 17.5 |
| 15-Sergey Brin (\$ 16.7 Billion)-Google | 5 | 16.7 |
| 15 - Larry Page (\$ 16.7 Billion)-Google | 5 | 16.7 |
| 18 - Michael Dell (\$ 15 Billion) - Dell | 5 | 15.0 |
| 19 -Steve Ballmer (\$ 13.9 Billion)-Microsoft | 5 | 13.9 |
| 23-Paul Allen (\$ 13.2 Billion)-Microsoft | $\frac{5}{5}$ | 13.2 |
| 38 - James Goodnight (\$7.1 Billion) - 5 AS Institute | 5 | 7.1 |
| 39 - Steve Jobs (S 7 Billion) - Apple, Pixar | S | 7.0 |
| 50 - Pierre Omidyar (\$6.2 Billion)-Ebay | 5 | 6.2 |
| 50 - Eric Schmidt (\$ 6.2 Billion)-Google | 5 | 6.2 |
| 86 - Gordon Moore (\$ 3.7 Billion) - Intel | 5 | 3.7 |
| 91 - Dustin Moskovitz (\$ 3.5 Billion)-Facebook | 5 | 3.5 |
| 91 -H. Poss Perot ( $\$ 3.5$ Billion) - Computer Servic | 5 | 3.5 |
| 91 - John Sall (\$ 3.5 billion) - SAS Institute | 5 | 3.5 |

## Income inequality is worse among the 400 richest Americans:

Bill Gates earns $30 \times$ median in a population of 400 !


## Wealth

## NATIO <br> The Rich, the Poor And the Oval Office

Being wealthy is no bar to the presidency. The trick is to convince voters that you can feel their pain
BY RICHARD NORTON SMITH

Rman-the American presidency has had its share
of each. Yet history us that economic status sone of the less reliable lead erformance The pffice $h a s$ en occupied by lass warriors and self $m$ ene worshippers of sitalist ma. Indeed, the Oval Office may be one place where size doesn't matter. At least where the size of one's fortune is oncerned. It's how a President defines success and what, if any, scars he has accumulated in his rise to power that reveal more than his net worth
It's hard to imagine Abraham Lincoln as a traitor to his class. But as a candidate or Congress in 1843, Lincoln was the target of a whispering campaign inspired by his marriage into the elite Todd and Edwards families. Lincoln professed astonishment that anyone who had known him
sa sio-d-month flatboat man could really believe he had morphed into "the candidate of pride, wealth and aristocratic family distinction." Likewise, his legal work for the Illinois Central Railroad-for which he received as much as $\$ 5,000$ case-would laterbe cited as fiend of the Lincoln wasno
He inoculated himself.
He than rinst such charges in the pring of 860 , when Lincoln for-President partisins marched into the lllinois Republican state convention bearing a weathered pair of fence rails purportedly spli by their hero. In another bit by their hero. In another bit the candidate was called upor to acknowledge his youthful handiwork. He wasn't certain of the rails in question, said incoln, but he had split a ood many better ones since growing to manhood. In that moment, A. Lincoln, Railroad awyer, gave way to Abe Lincoln, Rail Splitter, a more
marketable image for one who hadn't betrayed his origins so much as he had spent a life ime escaping them. Mitt Romney says Americans celebrate success, and he's right. But that doesn't mean they do so uncritically, still less that they agree on its meaning. The controversy surrounding Romney's nd whewit was acquired ase whor he understands cose who have been less accessuris partor a long rain and public obligation It sinseparable from presi.f ial politics, in which the big est single determinat of any incumbent's chances for re. lection is usually his record feconomic stewardship. No matter the era, what asn't changed is the vulnera bility of politicians in times of conomic distress. No one berudged the first George Bush his ancestral home in Kennebunkport, so reminiscent of the Kennedy compound down
the coast in Hyannis Port. There the parallels end. That Kennedy lived off a sto million trust fund established by his bootlegging father didn't preclude him from labeling price-gouging steel executives sons of bitches." Like Theo dore Roosevelt confronting "malefactors of great wealth" as embodied by J.P. Morgan, Kennedy defined himself not by his money but by his nemies. The gentlemanly While IFK's no sailing fights. wile Kissailingprowess elassy Bushsigend or classy guy, Bush'scigarette overhyped encounter with a supermarket scanner fed the narrative of country squire with oil on his boots his instincts dulled by noblesse oblige. Would it have gener ated the same reaction had the country not been in recession? The question answers itself JFK was right. Life is unfair. Few questioned the tax ruling under which Dwigh

Eisenhower, World War II's ultimate hero, was able to reat the considerable income from his war memoirs as a capital gain. Even the IRS liked Ike. By contrast, Herbe Hoover, having earned mil lions as the Great Enginee and given away much of it sfused his salary That didn't ep him from becoming a keep him from becoming a arture of Depression-ra a batwing collar.

Through the smoke screen f faux populism, one can discern the real issue at stake Is a President-or would-be President-out of touch? Pov erty, it seems, may or may not generate sympathy, but it's no guarantee that the man in the Oval Office will be seen as one of us. During his first week on the job, Gerald Ford asked a White House staffer when he could expect the first installment of his presidential salary He had a son about to enroll in college, Ford explained; a quarter-century on Capitol Hill had done nothing for the family finances. That the Fords were among millions of American families living paycheck to paycheck failed to establish a Trumanesque bond between him and other of modest means. Self-made Ford may have been, but he was never self-dramatizing.

Symbolism matters. Long before Jimmy Carter rebuked the imperial presidency by walking down Pennsylvania

How the GOP Candidates Stack Up
Like many Presidents before them, each has a fortune-one
is just larger than the rest. Here's how they made their money is just larger than the rest. Here's how they made their money


Mitt Romney $\$ 220$ million
As CEO of Bain Capital, Romney made a living
turning around struggling companies
Newt Gingrich $\$ 7$ million
After 20 years in Congress, he turned to

Ron Paul $\$ 4$ million
The Representative wrote best-selling books after a 30 -year career as an obstetrician

Rick Santorum \$1 million
The former Senator became a consultant and The former Senator became a consultant and
media personality after his tenure in Congress

Avenue on his Inaugural Day, Thomas Jefferson dispensed with his predecessors' horsedrawn coaches for his oath taking. The epicurean Jefferson was as tightfisted with the people's money as he was reckless in spending his own. As President, he slashed the military budget and shuttered foreign outposts in his determination to liquidate the national debt. "It is to be regretted that the rich and powerful too often bend he acts of government to heir selfish purposes," Andrew Jackson wrote in vetoing the rechartering of a national bank. Only a President, implied Jackson a slave master and the owner of a large plantation),
can protect the humbler mem bers of society from monopoly and crass exploitation. Theodore Roosevelt's Square Deal, Franklin Roosevelt's New Deal and Harry Truman's Fair Deal were not conceived in the spirit of us vs. them. Rather, each was envisioned as a chapter in the unfolding story of American democracy, con frming the irrelevance accounts alike

Put another way, fortune may favor the $\mathrm{I} \%$, but history rewards those who redistribute opportunity. As in other periods of economic distress, the 2012 election promises a referendum on the Forgotten Man. Is he the victim
of capitalist redundancy and Wall Street ruthlessness to whom FDR brilliantly pitched his political revolution? Or is
he the patriot at war with the popular culture, feeling his talents oppressed by Washands 'spress wh ngton's social engineers, who 68? Nion's brand of loth ar at conservatismanio arn's ren its populist ffshot, the Tea Party shoot, the Tea Party
Ultimately, what's in a Pre dent's bank account matters voter's. A century ago, amid he nation's worst depression -date, William McKinley as "the advance agent of posperity" just three years fter he flirted with personal ankruptcy. Rich friends ailed him out of his distress heir patronage didn't hurt McKinley's chances among n electorate radicalized by conomic despair. Counter intuitive as it seems, is conpicuous success anymore a disqualifier? Might a quarter billionaire with millions stashed in the Cayman Islands defy the odds by making him self the candidate of today's Forgotten Man? The answer epends largely on how voter gauge their prospects come November. In the meantime word to the wise of both parties: Stay away from speed oats and Donald Trump.
mith is a scholar-in-residence at George Mason University

John F. Kennedy
Kennedy personally lived off a $\$ 10$ million trust
fund, drawing from a family fortune built on rea
estate, investments and, allegedly, bootlegsing

$\$ 125$
George W. Bush
He started in the oil business and
later bought the Texas Range
baseball team with partners Bill Clinton
After leaving office, he earned a

0After leaving office, he earned a
$\$ 12$ million advance for his 2001 memoir, My Life, and millions more for speaking engagements -


598

## Wealth



Left graph shows how $90 \%$ of a population follows a log-normal wealth distribution, while the richest $10 \%$ veers off in a tail following a Pareto power law distribution.

Power-law Distribution $\rho(w) \propto w^{-\gamma}, w \in\left(w_{1}, w_{2}\right)$
$w$ is extensive quantity (additive: $w_{\mathrm{AB}}=w_{\mathrm{A}}+w_{\mathrm{B}}$ ) dollar, land, citation, degree of connection, energy, ...

## In addition to wealth, other examples include

- Earthquakes (energy)
- Nuclear Accidents (damage)
- War and Terrorism (casualty)
- Languages (Zipf’s law)
- Geometry


## Earthquakes

## Property damages caused by earthquake

| Rank | Name | Magnitude | Property damages |
| :---: | :--- | :--- | :--- |
| $\mathbf{1}$ | 2011 Tōhoku earthquake, Japan | $9.0^{[27]}$ | $\$ 122$ billion $^{[28]}$ |
| $\mathbf{2}$ | 1995 Great Hanshin earthquake, Japan | 6.9 | $\$ 100$ billion $^{[29]}$ |
| $\mathbf{3}$ | 2008 Sichuan earthquake, China | 8.0 | $\$ 75$ billion $^{[29]}$ |
| $\mathbf{4}$ | 2010 Chile earthquake, Chile | 8.8 | $\$ 15-30$ billion $^{[30]}$ |
| $\mathbf{5}$ | 1994 Northridge earthquake, United States | 6.7 | $\$ 20$ billion |
| $\mathbf{6}$ | 2011 Christchurch earthquake, New Zealand | $6.3^{[31]}$ | $\$ 12$ billion |
| $\mathbf{7}$ | 1989 Loma Prieta earthquake, United States | $6.9^{[32]}$ | $\$ 11$ billion |
| $\mathbf{8}$ | 921 earthquake, Taiwan $(1999)$ | 7.6 | $\$ 10$ billion |
| $\mathbf{9}$ | 1906 San Francisco earthquake, United States | $7.9^{[33]}$ | $\$ 9.5$ billion |
| $\mathbf{1 0}$ | 1960 Valdivia earthquake, Chile | $9.5^{[34]}$ | $\$ 2.9-5.8$ billion |



Time history of radiated energy from earthquakes throughout all of 1995. Sethna, Dahmen, Myers, "Crackling noise," Nature 410 (2001) 242.

## Wealth

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Being wealthy is no bar to the presidency. The trick is to convince voters that you can feel their pain
BY RICHARD NORTON SMITH

Rman-the American presidency has had its share
of each. Yet history us that economic status sone of the less reliable lead erformance The pffice $h a s$ en occupied by lass warriors and self $m$ ene worshippers of sitalist ma. Indeed, the Oval Office may be one place where size doesn't matter. At least where the size of one's fortune is oncerned. It's how a President defines success and what, if any, scars he has accumulated in his rise to power that reveal more than his net worth
It's hard to imagine Abraham Lincoln as a traitor to his class. But as a candidate or Congress in 1843, Lincoln was the target of a whispering campaign inspired by his marriage into the elite Todd and Edwards families. Lincoln professed astonishment that anyone who had known him
sa sio-d-month flatboat man could really believe he had morphed into "the candidate of pride, wealth and aristocratic family distinction." Likewise, his legal work for the Illinois Central Railroad-for which he received as much as $\$ 5,000$ case-would laterbe cited as fiend of the Lincoln wasno
He inoculated himself.
He than rinst such charges in the pring of 860 , when Lincoln for-President partisins marched into the lllinois Republican state convention bearing a weathered pair of fence rails purportedly spli by their hero. In another bit by their hero. In another bit the candidate was called upor to acknowledge his youthful handiwork. He wasn't certain of the rails in question, said incoln, but he had split a ood many better ones since growing to manhood. In that moment, A. Lincoln, Railroad awyer, gave way to Abe Lincoln, Rail Splitter, a more
marketable image for one who hadn't betrayed his origins so much as he had spent a life ime escaping them. Mitt Romney says Americans celebrate success, and he's right. But that doesn't mean they do so uncritically, still less that they agree on its meaning. The controversy surrounding Romney's nd whewit was acquired ase whor he understands cose who have been less accessuris partor a long rain and public obligation It sinseparable from presi.f ial politics, in which the big est single determinat of any incumbent's chances for re. lection is usually his record feconomic stewardship. No matter the era, what asn't changed is the vulnera bility of politicians in times of conomic distress. No one berudged the first George Bush his ancestral home in Kennebunkport, so reminiscent of the Kennedy compound down
the coast in Hyannis Port. There the parallels end. That Kennedy lived off a sto million trust fund established by his bootlegging father didn't preclude him from labeling price-gouging steel executives sons of bitches." Like Theo dore Roosevelt confronting "malefactors of great wealth" as embodied by J.P. Morgan, Kennedy defined himself not by his money but by his nemies. The gentlemanly While IFK's no sailing fights. wile Kissailingprowess elassy Bushsigend or classy guy, Bush'scigarette overhyped encounter with a supermarket scanner fed the narrative of country squire with oil on his boots his instincts dulled by noblesse oblige. Would it have gener ated the same reaction had the country not been in recession? The question answers itself JFK was right. Life is unfair. Few questioned the tax ruling under which Dwigh

Eisenhower, World War II's ultimate hero, was able to reat the considerable income from his war memoirs as a capital gain. Even the IRS liked Ike. By contrast, Herbe Hoover, having earned mil lions as the Great Enginee and given away much of it sfused his salary That didn't ep him from becoming a keep him from becoming a arture of Depression-ra a batwing collar.

Through the smoke screen f faux populism, one can discern the real issue at stake Is a President-or would-be President-out of touch? Pov erty, it seems, may or may not generate sympathy, but it's no guarantee that the man in the Oval Office will be seen as one of us. During his first week on the job, Gerald Ford asked a White House staffer when he could expect the first installment of his presidential salary He had a son about to enroll in college, Ford explained; a quarter-century on Capitol Hill had done nothing for the family finances. That the Fords were among millions of American families living paycheck to paycheck failed to establish a Trumanesque bond between him and other of modest means. Self-made Ford may have been, but he was never self-dramatizing.

Symbolism matters. Long before Jimmy Carter rebuked the imperial presidency by walking down Pennsylvania

How the GOP Candidates Stack Up
Like many Presidents before them, each has a fortune-one
is just larger than the rest. Here's how they made their money is just larger than the rest. Here's how they made their money


Mitt Romney $\$ 220$ million
As CEO of Bain Capital, Romney made a living
turning around struggling companies
Newt Gingrich $\$ 7$ million
After 20 years in Congress, he turned to

Ron Paul $\$ 4$ million
The Representative wrote best-selling books after a 30 -year career as an obstetrician

Rick Santorum \$1 million
The former Senator became a consultant and The former Senator became a consultant and
media personality after his tenure in Congress

Avenue on his Inaugural Day, Thomas Jefferson dispensed with his predecessors' horsedrawn coaches for his oath taking. The epicurean Jefferson was as tightfisted with the people's money as he was reckless in spending his own. As President, he slashed the military budget and shuttered foreign outposts in his determination to liquidate the national debt. "It is to be regretted that the rich and powerful too often bend he acts of government to heir selfish purposes," Andrew Jackson wrote in vetoing the rechartering of a national bank. Only a President, implied Jackson a slave master and the owner of a large plantation),
can protect the humbler mem bers of society from monopoly and crass exploitation. Theodore Roosevelt's Square Deal, Franklin Roosevelt's New Deal and Harry Truman's Fair Deal were not conceived in the spirit of us vs. them. Rather, each was envisioned as a chapter in the unfolding story of American democracy, con frming the irrelevance accounts alike

Put another way, fortune may favor the $\mathrm{I} \%$, but history rewards those who redistribute opportunity. As in other periods of economic distress, the 2012 election promises a referendum on the Forgotten Man. Is he the victim
of capitalist redundancy and Wall Street ruthlessness to whom FDR brilliantly pitched his political revolution? Or is
he the patriot at war with the popular culture, feeling his talents oppressed by Washands 'spress wh ngton's social engineers, who 68? Nion's brand of loth ar at conservatismanio arn's ren its populist ffshot, the Tea Party shoot, the Tea Party
Ultimately, what's in a Pre dent's bank account matters voter's. A century ago, amid he nation's worst depression -date, William McKinley as "the advance agent of posperity" just three years fter he flirted with personal ankruptcy. Rich friends ailed him out of his distress heir patronage didn't hurt McKinley's chances among n electorate radicalized by conomic despair. Counter intuitive as it seems, is conpicuous success anymore a disqualifier? Might a quarter billionaire with millions stashed in the Cayman Islands defy the odds by making him self the candidate of today's Forgotten Man? The answer epends largely on how voter gauge their prospects come November. In the meantime word to the wise of both parties: Stay away from speed oats and Donald Trump.
mith is a scholar-in-residence at George Mason University

John F. Kennedy
Kennedy personally lived off a $\$ 10$ million trust
fund, drawing from a family fortune built on rea
estate, investments and, allegedly, bootlegsing

$\$ 125$
George W. Bush
He started in the oil business and
later bought the Texas Range
baseball team with partners Bill Clinton
After leaving office, he earned a

0After leaving office, he earned a
$\$ 12$ million advance for his 2001 memoir, My Life, and millions more for speaking engagements -


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Number of earthquakes on Earth in 1995 exceeding Richter magnitude $M$

Gutenberg-Richter law $N($ magnitude $>M) \approx 10^{a-b M}$


Probability vs. released Radioactive materials $1 \mathrm{Ci}=3.7 \times 10^{10}$ decays per second $\sim 1$ gram of radium ${ }^{226} \mathrm{Ra}$ http://www.asahi-net.or.jp/~pu4i-aok/cooldata2/politics/fukushimameltdown.htm


Neil F. Johnson et al, "Universal patterns underlying ongoing wars and terrorism," http://xxx.lanl.gov/abs/physics/0506213

Events before May 1st, 2003

| Start Date | City | Killings Min. | Killings Max |
| :---: | :---: | :---: | :---: |
| 3/20/03 | Baghdad | 1473 | 2000 |
| 3/20/03 | Najaf | 224 | 358 |
| 3/20/03 | Basra | 142 | 200 |
| 3/20/03 | Najaf, Karba, Mosul, Samawa, Madain, Diwaniyah, Kut, | 484 | 445 |
| 3/20/03 | Baghdad | 22 | 22 |
| 3/20/03 | Rutba | 1 | 1 |
| 3/21/03 | Baghdad | 0 | 3 |
| 3/21/03 | Umm Qasr | 2 | 2 |
| 3/22/03 | ImAnas | 1 | 1 |
| 3/22/03 | Mosul | 4 | 4 |
| 3/22/03 | Nassiriya | 12 | 12 |
| 3/22/03 | Basra | 50 | 77 |
| 3/22/03 | Tikrit | 4 | 5 |
| 3/22/03 | Kurdistan | 57 | 100 |
| 3/23/03 | Najaf | 3 | 8 |
| 3/23/03 | Rutbah | 5 | 5 |
| 3/23/03 | Babel | 30 | 30 |
| 3/23/03 | Basra | 14 | 14 |
| 3/23/03 | Karba | 10 | 10 |
| 3/23/03 | Nassiriya | 10 | 10 |
| 3/24/03 | Baghdad | 5 | 5 |
| 3/24/03 | Baghdad | 5 | 5 |
| 3/25/03 | Ash Shatra | 2 | 2 |
| 3/25/03 | Nassiriya | 2 | 2 |
| 3/26/03 | Rutbah | 2 | 2 |
| 3/26/03 | Baghdad | 14 | 14 |
| 3/26/03 | Baghdad | 21 | 21 |
| 3/27/03 | Missan | 2 | 2 |
| 3/27/03 | Mosul | 2 | 50 |
| 3/27/03 | Waset | 2 | 2 |
| 3/27/03 | Baghdad | 7 | 7 |
| 3/27/03 | Babel | 26 | 26 |
| 3/27/03 | Karba | 11 | 11 |
| 3/27/03 | Hillah | 78 | 201 |
| 3/27/03 | Najaf | 26 | 26 |
| 3/28/03 | Baghdad | 34 | 62 |
| 3/28/03 | Anbar | 28 | 28 |
| 3/28/03 | Babel | 3 | 3 |
| 3/28/03 | Baghdad | 6 | 6 |
| 3/28/03 | Karba | 6 | 6 |
| 3/28/03 | Najaf | 35 | 35 |
| 3/29/03 | Unknown | 1 | 1 |

Events before May 1st, 2003

| Start Date |  | City | Killings Min. | Killings Max |
| :---: | :---: | :---: | :---: | :---: |
| 3/29/03 | Janabiin |  | 20 | 20 |
| 3/30/03 | Baghdad |  | 15 | 15 |
| 3/31/03 | Baghdad |  | 6 | 6 |
| 3/31/03 | Mosul |  | 21 | 21 |
| 3/31/03 | Hillah |  | 15 | 15 |
| 3/31/03 | Hillah |  | 24 | 24 |
| 3/31/03 | Najaf and Karba |  | 11 | 11 |
| 3/31/03 | Baghdad |  | 24 | 24 |
| 4/1/03 | Baghdad |  | 1 | 1 |
| 4/1/03 | Shatra |  | 1 | 1 |
| 4/1/03 | Hillah |  | 33 | 33 |
| 4/2/03 | Baghdad |  | 43 | 43 |
| 4/2/03 | Baghdad |  | 5 | 5 |
| 4/3/03 | Baghdad |  | 10 | 16 |
| 4/3/03 | Baghdad |  | 27 | 27 |
| 4/3/03 | Basra |  | 42 | 51 |
| 4/3/03 | Karba |  | 5 | 5 |
| 4/3/03 | Najaf |  | 0 | 40 |
| 4/4/03 |  |  | 17 | 17 |
| 4/4/03 | najaf |  | 7 | 7 |
| 4/4/03 | Baghdad |  | 6 | 6 |
| 4/5/03 | Karba |  | 1 | 1 |
| 4/5/03 | Baghdad |  | 22 | 22 |
| 4/5/03 | Rashidiya |  | 85 | 85 |
| 4/5/03 | Basra |  | 17 | 17 |
| 4/6/03 | irbil |  | 1 | 1 |
| 4/6/03 | Baghdad |  | 15 | 15 |
| 4/6/03 | karbala |  | 35 | 35 |
| 4/7/03 | Baghdad |  | 2 | 2 |
| 4/7/03 | Baghdad |  | 9 | 14 |
| 4/7/03 | Baghdad |  | 11 | 11 |
| 4/7/03 | Baghdad |  | 4 | 4 |
| 4/7/03 | Baghdad |  | 3 | 3 |
| 4/8/03 | Baghdad |  | 1 | 1 |
| 4/8/03 | Baghdad |  | 2 | 2 |
| 4/8/03 | Baghdad |  | 35 | 35 |
| 4/8/03 | Baghdad |  | 13 | 13 |
| 4/9/03 | Baghdad |  | 2 | 2 |
| 4/9/03 | Fathlia |  | 4 | 4 |
| 4/9/03 | Baghdad |  | 5 | 21 |
| 4/9/03 | Baghdad |  | 21 | 26 |
| 4/10/03 | Baghdad |  | 30 | 30 |
| 4/10/03 | Kirkuk |  | 40 | 40 |

## Zipf’s law

$\log$ (occurrence frequency)


## Fractal : Power-law distribution

Geometry


## How Long Is the Coast of Britain?

## Statistical Self-Similarity and Fractional Dimension

Abstract. Geographical curves are so involved in their detail that their lengths
Geometry are often infinite or, rather, undefinable. However, many are statistically "selfsimilar," meaning that each portion can be considered a reduced-scale image of the whole. In that case, the degree of complication can be described by a quantity D that has many properties of a "dimension," though it is fractional; that is, it exceeds the value unity associated with the ordinary, rectifiable, curves.

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## Materials Science




FIG. 2. Analysis of the cell structure in Fig. 1 by determining the "box dimension" $D_{\mathrm{B}}$, for details see text.

Hahner, Bay, Zaiser, "Fractal dislocation patterning during plastic deformation," Phys. Rev. Lett. 81 (1998) 2470

## Box-count scaling



| (0,1) |
| :--- |
|  |

## Box-count scaling



| (0,1) Box-count scaling |
| :--- |
|  |

## Fractal Dimension $f$

- Simple Line: box count $N \propto s^{-1}$
- Simple rectangle set: box count $N \propto s^{-2}$
- Not so-simple, but self-similar point set: box count $N \propto s^{-f}$


Koch snowflake
$f=1.2619$.

## Nanofactory ${ }^{\mathrm{TM}}$ TEM-STM

Center for Integrated Nanotechnologies (CINT)
@ Sandia National Lab


Joule heated to $\sim 2000{ }^{\circ} \mathrm{C}$ to induce sublimation
Tecnai F30 @ 300 kV





Fig. S2. Fractal dimension measurement procedures. The fractal dimension of the propagating front pattern was measured by the same method as that used to measure the coastline [Sapoval B, Baldassarri A, Gabrielli A (2004) Phys Rev Lett 93:098501]. First, a close loop was drawn by fitting the experimental propagating sublimation front (Fig. S2 A and B). Then the close loop was measured on a 2D square lattice. In the measurement, only those squares that intersect with the close loop were accounted (e.g., the gray squares in Fig. S 2 C ). By varying the size of the squares, the number of accounted squares as a function of a square size (i.e., the measuring unit) was plotted (Fig. S2D). According to the definition of the fractal dimension, the fractal dimension of the loop or the propagating loop is calculated as:

$$
d=-\frac{d \log (N)}{d \log (I)}
$$

Benoit Mandelbrot, "How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension." Science 156 (1967) 636.


## Sierpinski Triangle: Fractal dimension 1.5849625

## Sierpinski Hexagon: Fractal dimension 1.6309297

In this lecture, show connections between:

1. Power-law distribution ("scale-free" behavior)
2. "Network science" model: growth and preferential attachment
3. Often, hierarchical organization of society and nature diameter of internet $\approx 20$


## A real-space network



To go from one vertex to another, takes many bond hops


Social Network, WWW, Genetic Network, ... "Small-World Network" problem with this terminology: "small-world network" is not small, but often huge.
Humans on earth $\left(7 \times 10^{9}\right)$ form a relationship network of diameter 6 .

## $\langle d\rangle=0.35+2.06 \log _{10} N$ clicks from go from any document to any other document in a $N$-vertex WWW.




Figure 1 Distribution of links on the World-Wide Web. a, Outgoing links (URLs found on an HTML document); $\mathbf{b}$, incoming links (URLs pointing to a certain HTML document). Data were obtained from the complete map of the nd.edu domain, which contains 325,729 documents and 1,469,680 links. Dotted lines represent analytical fits used as input distributions in constructing the topological model of the web; the tail of the distributions follows $P(k) \approx k^{-\gamma}$, with $\gamma_{\text {out }}=2.45$ and $\gamma_{\mathrm{n}}=2.1$. c, Average of the shortest path between two documents as a function of system size, as predicted by the model. To check the validity of our predictions, we determined $d$ for documents in the domain nd.edu. The measured $\left\langle d_{\text {nd.edu }}\right\rangle=11.2$ agrees well with the prediction $\left\langle d_{3 \times 10^{5}}\right\rangle=11.6$ obtained from our model. To show that the power-law tail of $P(k)$ is a universal feature of the web, the inset shows $P_{\text {out }}(k)$ obtained by starting from whitehouse.gov (squares), yahoo.com (triangles) and snu.ac.kr (inverted triangles). The slope of the dashed line is $\gamma_{\text {out }}=2.45$, as obtained from nd.edu in $\mathbf{a}$.

[^0]

Fig. 1. The distribution function of connectivities for various large networks. (A) Actor collaboration graph with $N=212,250$ vertices and average connectivity $\langle k\rangle=28.78$. (B) WWW, $N=$ $325,729,\langle k\rangle=5.46$ (6). (C) Power grid data, $N=4941,\langle k\rangle=2.67$. The dashed lines have slopes $(A) \gamma_{\text {actor }}=2.3$, $(B) \gamma_{w w w}=2.1$ and $(C) \gamma_{\text {power }}=4$.

Barabasi, Albert, "Emergence of scaling in random networks," Science 286 (1999) 509-512. \%6316 cites

## Small-World Networks

- Data, web, business associations: Relationship, rather than real-space physical / chemical attachment
- not strongly exclusive (unlike steric repulsion in chemical bonding, or marriage), possible to develop huge $k$
- Long tails: Scale-free power-law distribution
- Vital few: like hubs, shrinks the network diameter
- Connectivity is power

Mb $=$ megabit $=1,000,000$ bit, $M B=$ megabyte $=$ 8Mb

## Mbps $=$ megabit $/$ second $=\mathbf{0 . 1 2 5} \mathrm{MB} / \mathrm{s}=\mathbf{1 2 5} \mathrm{kB} / \mathrm{s}$

http://www.speedtest.net/
Getting 1 Mbps in travel is pretty decent connection
But one can live with maybe $0.1 \mathrm{Mbps} \sim 13 \mathrm{kB} / \mathrm{s}$
From MIT office copper line, download: 6.5 Mbps , upload 7.7 Mbps ~ 1 MB/s Upper limit: Ethernet 10 Mbps , fast Ethernet 100Mbps, gigabit Ethernet 1000Mbps

$$
\begin{gathered}
1 \mathrm{~GB}=1 \text { gigabyte }=1,000,000,000 \text { byte } \\
1 \text { gibibyte }=1024^{3} \text { bytes }=1,073,741,824 \text { byte }
\end{gathered}
$$

64-bit machines can address $18,446,744,073$ GB memory

| C type | ILP64 (Cray) | LP64 (MacOS X, <br> Linux) | LLP64 <br> (Windows) |
| :---: | :---: | :---: | :---: |
| char | 8 | 8 | 8 |
| short | 16 | 16 | 16 |
| int | 64 | 32 | 32 |
| long | 64 | 64 | 32 |
| long long | 64 | 64 | 64 |
| pointer | 64 | 64 | 64 |

## Gini coefficient $\equiv \mathrm{A} /(\mathrm{A}+\mathrm{B})$

Distribution of world GDP, 1989

| Quintile of Population | Income |
| :--- | :--- |
| Richest 20\% | $82.7 \%$ |
| Second 20\% | $11.7 \%$ |
| Third 20\% | $2.3 \%$ |
| Fourth 20\% | $1.4 \%$ |
| Poorest 20\% | $1.2 \%$ |

SOURCE: WIKIPIDEA


Cumulative share of people from lowest to highest incomes
from Wikipedia

Gini Index - Income Disparity since World War II
where 0 is perfect equality, and 100 is perfect inequality (i.e., one person has all the income)



[^0]:    Albert, Jeong, Barabasi, "Internet - Diameter of the World-Wide Web," Nature 401 (1999) 130. \% 1382 cites

