Anisotropic hardening equations derived from reverse-bend testing

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Abstract

The plastic response of materials during reverse loading has practical consequences for common sheet forming operations in terms of loads, localization behavior, and springback. However, it is difficult to measure the reverse loading (Bauschinger effect) in sheet materials because of their propensity to buckle. A simple reverse-bend test was constructed and used to investigate the cyclic loading of three automotive body alloys. The results showed that consideration of the Bauschinger effect is essential to obtaining agreement with such results. An inverse procedure was used to determine anisotropic hardening law parameters. Laws obtained in this way were compared with ones generated by more sensitive tension-compression tests appearing in the literature for the same alloys. The two laws were significantly different, but both produced accurate simulations of reverse-bend test load–displacement curves. Several artificial material models were then constructed to simulate the reverse-bend test and thus to probe its sensitivity to material constitutive equation details. For materials whose reverse-loading response varies with the level of prestrain, as is the case for each of the three alloys tested, a wide range of constitutive response is capable of producing identical reverse-bending behavior. The results show that inverse procedures applied to the reverse-bend test do not provide unique results, and thus the usefulness of the reverse-bend test for such investigations is limited. © 2002 Published by Elsevier Science Ltd.

Keywords: Cyclic plasticity; Reverse bending; Tension–compression; Anisotropic hardening; Aluminum; Steel

1. Introduction

In many sheet metal forming processes, sheet specimens can experience cyclic loading conditions. The hardening behavior of materials is more complicated under
such conditions than under monotonic loading. For materials that exhibit a Bauschinger effect, the yield stress in reverse loading is usually lower than in the case of continuous loading, and the subsequent hardening rate is higher. In such cases, the conventional isotropic hardening model is no longer an adequate approximation. Therefore, for some materials under reverse loading, consideration of the Bauschinger effect is required for a realistic stress distribution, which is in turn essential for prediction of springback under bending/unbending conditions (Li et al., 1999b; Geng, 2000; Geng and Wagoner, 2000).

The Bauschinger effect refers to a decrease of flow stress upon a stress reversal (Sowerby et al., 1979; Dafalias, 1984; Abel, 1987; Mughrabi, 1987), although in some ferrous alloys the flow stress may increase (Ghosh and Backofen, 1973; Laukonis and Ghosh, 1978; Wagoner and Laukonis, 1983). Measurement of the effect is usually done under cyclic torsion conditions for bulk materials (Svensson, 1966; Hecker, 1971, 1973; Lipkin and Swaarengen, 1975; Anand and Gurland, 1976; Chang and Asaro, 1978; Drucker and Palgen, 1981), although reverse shear tests (Miyauchi, 1984a,b; Vreede, 1992) and in-plane reverse uniaxial tests (Hoge et al., 1973; Tan et al., 1994; Kuwabara et al., 1995) have also been presented for sheet or plate materials. A special version of the in-plane tension–compression test was devised (Balakrishnan, 1999) for sheet materials to avoid buckling for compressive strains up to 0.08. Corrections were required for biaxiality and friction introduced by the fixture (Balakrishnan, 1999). A reverse-bend test (Jiang, 1997; Shen, 1999; Zhao, 1999; Zhao and Lee, 1999) provides a simple alternative for testing sheet metals under reverse loading, and has been proposed for measuring the Bauschinger effect in sheet materials, but it does not provide direct stress–strain data.

To model the material hardening behavior revealed by these tests, anisotropic phenomenological models are needed, various forms of which have been proposed. Classical kinematic hardening models (Prager, 1956; Ziegler, 1959) have simple formulations with sharp reverse yield at fixed stresses. Hodge’s mixed hardening model (Hodge, 1957) reproduces the permanent softening upon reverse loading, again without transient yield or hardening. Multilayer (Besseling, 1958) and multiyield surface (Mroz, 1967, 1969) models reproduce transient aspects at the expense of many fit parameters, and even more complex formulations have appeared (Baltov and Sawczuk, 1965; Williams and Svensson, 1971; Rees, 1981).

In two-surface models, the yield surface translates and expands within an enclosing bounding surface. A set of differential equations is defined to describe the translation and expansion of these surfaces (Krieg, 1975), or the plastic modulus can be defined as a function of the distance between the two (Dafalias and Popov, 1976). An update procedure is essential whenever a drastic change in loading direction occurs. The nonlinear-kinematic hardening model (Armstrong and Frederick, 1966; Chaboche, 1986) is a generalization of Prager and Ziegler’s linear kinematic hardening model: a recall term is introduced into the classical kinematic hardening formulation to reproduce the smooth elastic-plastic transition upon the change of loading path. Reviews of such models have been presented (Chaboche, 1986; Khan and Huang, 1995).

The nonlinear kinematic hardening model was fit to published in-plane tension–compression test results because it approximates faithfully the nonlinear stress–strain
curves in the complex loading paths, and it is readily implemented. The two material parameters in the model can be defined as functions of prestrain, in accord with detailed measurements (Balakrishnan, 1999).

The current work focuses on the consistency of anisotropic hardening laws derived from tension–compression and reverse-bending tests, and on the uniqueness of laws determined from fitting reverse-bend tests via an inverse procedure. The Chaboche law (Chaboche, 1986) constants are first fit to tension–compression test curves. The best-fit parameters are found to vary markedly with prestrain, i.e. the strain at the stress reversal. Next, a similar fit is performed to reverse-bend data, with least-squares Chaboche constants found by a trial-and-error inverse optimization. Finally, the constitutive equation obtained from tension-compression tests is used to simulate the reverse-bend test. As will be shown, the laws obtained in these alternate ways fit the reverse-bend test equally well, but diverge when compared with tension–compression data. Therefore, the reverse-bend test cannot be used to measure the Bauschinger effect uniquely because the measured load–displacement points each involve a range of stresses and strains simultaneously.

2. Experimental procedures

2.1. Materials

Three automotive body alloys were tested and analyzed: 6022-T4 aluminum alloy, high-strength low-alloy steel (HSLA), and drawing-quality silicon-killed steel (DQSK). Chemical composition of the three alloys is listed in Table 1. 6022-T4 is an aluminum–magnesium–silicon heat-treatable alloy. The strength is derived from a high temperature solution heat treatment followed by a rapid quench and a precipitation aging (Mg2Si) treatment at room temperature. HSLA is a microalloyed and precipitation strengthened pearlite-ferritic as-rolled steel with Mn for sulfide control and strength, and Nb for carbide formation and grain refinement to increase strength. DQSK is a very low-carbon steel, similar to HSLA but with approximately half of the carbon and Mn, processed to obtain a high R-value (for improved deep drawability). The microstructures of the three alloys used have been presented (Balakrishnan, 1999). The average grain diameters of 6022-T4, HSLA and DQSK are 150, 35 and 10 microns, respectively. The uniaxial tensile flow curves for the three materials are shown in Fig. 1 (Balakrishnan, 1999).

<table>
<thead>
<tr>
<th>Chemical compositions of three alloys, in weight percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>DQSK</td>
</tr>
<tr>
<td>HSLA</td>
</tr>
<tr>
<td>6022-T4</td>
</tr>
</tbody>
</table>
2.2. The reverse-bend test

Reverse-bend tests were carried out using a specially-constructed three-point bending device (Fig. 2). The upper fixture or punch consists of two vertically-aligned, non-rotating rods in close contact with the center of the strip specimen. The lower fixture consists of two freely-rotating bearings into each of which are mounted four rotatable rollers, an upper pair and a lower pair. The lower pair of rollers is at a fixed position in the bearing while the upper pair may be adjusted vertically to accommodate materials of various thicknesses. The rollers are located such that a material 1 mm thick passes through the bearing centerline. A load cell is mounted above the punch to measure the axial force, and a computer records the punch displacement and force.

Shear-cut specimens and electrical discharge machined (EDM) specimens of nominally 25.4 mm width were tested. The EDM samples exhibited smaller experimental scatter than those of the shear cut specimens, but otherwise showed no differences. The
bending moment or load was normalized to nominal specimen width \( b_0 \) and thickness \( t_0 \) (Lange, 1985) to account for specimen-to-specimen scatter of these quantities, as follows:

\[
L_N = L \frac{b_0}{b} \left( \frac{t_0}{t} \right)^2
\]

where \( L, b, t \) are the measured load, width and thickness, respectively, and \( L_N \) is the normalized load.

Two configurations of rollers were compared. In one case, all four rollers were utilized in contact, in the other case only the ones opposing the punch force were used. No significant difference was found in the results from these two configurations, thus validating the use of an analytical model introduced in the next section.

Fig. 3 shows typical load–displacement curves generated for the three materials. The elastic range can be modeled as a cantilever beam to obtain the Young’s modulus from the initial linear slope of the load \((p)–\)displacement \((w)\) curve (Gere and Timoshenko, 1984; Jiang, 1997):

\[
E = \frac{4bl^3}{t^3} \left( \frac{dp}{dw} \right)
\]

where \( b, t \) and \( l \) are the width, thickness and distance between the centers of the two bearings, respectively. The slopes of initial and reverse load–displacement curves are shown in Fig. 3, each fit to lines with correlation coefficients greater than 0.9999. The Young’s moduli calculated according to Eq. (2), along with those from literature, are reported in Table 2. As is frequently observed, the moduli determined from

![Fig. 3. Typical bend/reverse-bend results for three materials. Initial slopes and proportional limits are shown.](image-url)
mechanical testing are lower than those measured sonically. The difference presumably lies in microplastic or anelastic effects.

The difference between the displacements at full load and after unloading is a measure of springback. The presence of the Bauschinger effect is demonstrated by the reverse proportional limit being much lower than that of the forward flow stress. Deviation from linearity during unloading is observed for DQSK and HSLA, indicating the presence of plastic deformation in the unloading process.

Although the present work focused on a single reverse cycle, a few multi-cycle bending tests were performed for the three materials. The bending and reverse-bending curves tend to saturate after two cycles for HSLA, four cycles for DQSK, and six cycles for 6022-T4.

### 3. Modeling of the reverse-bend test

Various models of the reverse-bend test were constructed using techniques developed for analysis of draw-bend springback (Wagoner et al., 1997; Li and Wagoner, 1998). In particular, attention was paid to element size, number of integration points through the thickness (shell elements), number of elements through the thickness (solid elements), and convergence and contact tolerances. SHEET-S (Wagoner et al., 1989; Saran et al., 1991; Wang and Wagoner, 1991a; Keum et al., 1992; Wang and Wagoner, 1991b) was used for two-dimensional simulations, both plane stress and plane strain; and ABAQUS (Anon., 1998) for three-dimensional simulations. The 2-D simulations with SHEET-S utilized either Bernoulli or Timoshenko beam elements with either plane stress or plane strain constraints.

The material models for initial bend simulations made use of von Mises yield functions and isotropic strain hardening models. For the DQSK and 6022-T4 materials, the hardening curves shown in Fig. 1 were employed. For HSLA, the negative slope associated with the yield point phenomenon introduced convergence problems. This softening is usually associated with the appearance of Lüders bands (Sinha, 1989). Because of this non-uniformity of strain, there is no direct way to extract the true local stress-strain curve from the macroscopic tensile test in this range (Kocks, 1981; Duncan et al., 1999a, b; 1993). For HSLA, the effective stress–strain curve was adjusted following a modification of the flow curve suggested by Kocks (1981) and Duncan et al. (1993, 1999a). This curve, shown in Fig. 4, also had convergence difficulties at the elastic-plastic transition,
so an intermediate extrapolation that matched experiments well was generated by trial and error. This curve, shown in Fig. 4, was used for subsequent analysis.

A cantilever beam bending model (i.e. one-half of the symmetric physical problem) was constructed, with the driving action at the end of the beam provided by contact with two rods matching those carrying the reaction forces in the experiment. The two rods were free to rotate about a common axis located between their centerlines. For reverse-bending, the rods are located on the opposite side of the specimen. This model was found to reproduce the boundary conditions adequately.

For two-dimensional simulations, half of the specimen was meshed (Fig. 5a); while for three-dimensional simulations, a quarter of the specimen was meshed to take advantage of both axes of symmetry (Fig. 5b). In view of previous experience with simulation of springback (Li and Wagoner, 1998; Li et al., 1999a, 2000), 51 integration points were used through the thickness with shell elements (ABAQUS

![Fig. 4. Alternative hardening curves in the yield point phenomenon region for HSLA.](image)

![Fig. 5. Finite element meshes for (a) 2-D beam and (b) 3-D shell elements. The meshes for 3-D solid elements are obtained by extending the 3-D shell mesh to nine layers in the thickness direction.](image)
S4R) and nine layers were used through the thickness with the solid elements (ABAQUS C3D8R). As shown in Fig. 6 for 6022-T4 aluminum alloy, both 3-D solid and shell elements reproduce the experimental bending results accurately, while the plane-strain beam elements over-predict the load and plane-stress elements under-predict it.

The origin of the difference between 2-D and 3-D simulations lies with the presence of anticlastic curvature (Yu and Zhang, 1996; Carden et al., 2000; Li et al., 2000), the secondary curvature that occurs because of differential lateral strain from the outer fiber to inner fiber of the sheet caused by the principal bending. Fig. 7 shows the simulated secondary curvature for the specimen under load and after being unloaded for a section 8 mm away from the symmetry plane of the model (bend center of the physical specimen). Anticlastic curvature is observed in the experiments as well, but measurement accuracy is poor because of the small specimen width and small magnitude of the secondary curvature.

4. Anisotropic hardening models

Throughout the current work, a modified nonlinear kinematic/isotropic hardening model is used to represent plastic hardening. The model is based on several developments appearing in the literature. Armstrong and Frederick (1966) added to the classical kinematic hardening model of Prager (1956) a recall term to reproduce observed transient hardening following an abrupt change of load path. Chaboche (1986) generalized this approach to include multiple evolution equations of the yield surface location ($\alpha$, also known as the back stress) and incorporated a classical isotropic hardening component as well. The update of the backstress in the Prager
equations takes place along the direction of plastic strain, a procedure which has since been shown (Ziegler, 1959) to give inconsistent results unless specially formulated (Khan and Huang, 1995). Therefore, in the current work, the Armstrong–Frederick–Chaboche approach for nonlinear kinematic/isotropic hardening is adopted, with the exception that the steady-state evolution of $\dot{C}_{11}$ is given by Ziegler (1959):

$$\frac{d}{C}_{11} = \frac{C}{C_{27}0} \left( \frac{C_{13}}{C_{11}} \right) \dot{\varepsilon}_p$$

where $\dot{\varepsilon}_p$ is the equivalent plastic strain rate; and $C$ and $\dot{C}_{13}$ are material parameters. $C_{27}0$ is the yield surface size, which is constant in a purely kinematic hardening model, but which is allowed to evolve with plastic strain in a combined kinematic-isotropic model (as is adopted here). $C$, $\sigma_0$, and $\gamma$ determine the plastic hardening rate following an abrupt change of loading path.

For a given yield function

$$F = f(\sigma - \alpha) - \sigma_0 = 0,$$

and the consistency condition is expressed as follows:

$$\frac{\partial f}{\partial \sigma} : \dot{\sigma} - \frac{\partial f}{\partial \sigma} : \dot{\sigma} - \frac{d\sigma_0}{d\varepsilon_p} \dot{\varepsilon}_p = 0$$

For simplicity, only the von Mises yield function is considered in the current work. Substituting (3) into (5), and using
\[
\frac{\partial f}{\partial \sigma} : (\sigma - \alpha) = \sigma_0
\]

gains

\[
\frac{\partial f}{\partial \sigma} : \dot{\sigma} - \left( C - \gamma \frac{\partial f}{\partial \sigma} : \alpha + \frac{\partial \sigma_0}{\partial \varepsilon_p} \right) d\varepsilon_p = 0
\] (6)

For uniaxial loading, the plastic modulus is expressed as follows:

\[
H_p = \frac{d\sigma}{d\varepsilon_p} = C - \gamma \alpha \text{sgn}(\sigma - \alpha) + \frac{\partial \sigma_0}{\partial \varepsilon_p}
\] (7)

where \( \text{sgn}(x) = 1 \) if \( x > 0 \), or \( \text{sgn}(x) = -1 \) if \( x < 0 \).

For the case of a stress reversal, the second term in Eq. (6) changes sign. The parameters \( C \) and \( \gamma \) at the time of the reversal may be determined with the use of the following relationships:

\[
C = \frac{H_f(\varepsilon_p) + H_r(\varepsilon_p)}{2} - \frac{d\sigma_0}{d\varepsilon_p},
\] (8)

\[
\gamma = \frac{H_r(\varepsilon_p) - H_f(\varepsilon_p)}{2\alpha}
\] (9)

where \( H_f \) and \( H_r \) are the plastic moduli for the forward and reverse loading at the moment of stress reversal, respectively. The yield surface size, \( \sigma_o \), and the back stress, \( \alpha \) in Eq. (9), may be found from the measured forward and reverse flow stresses:

\[
\sigma_f = \sigma_o + \alpha
\] (10)

\[
\alpha = \sigma_f - \sigma_o = \frac{\sigma_f - \sigma_r}{2} = \frac{\sigma_f + \sigma_r}{2}
\] (11)

where \( \sigma_f \) and \( \sigma_r \) are the forward and reverse flow/yield stresses at the reversal. The other quantity in Eq. (8), \( d\sigma_0/d\varepsilon_p \), may be approximated by \( \Delta\sigma_o/\Delta\varepsilon_p \), from stress reversals carried out at other prestrains.

A hardening law with similar formulation is available in the commercially available FEA code ABAQUS (Anon., 1998), where \( \gamma \) is required to be constant, and \( C \) may change with the temperature or field variables. To relax this constraint, a nonlinear kinematic hardening law with \( C \) and \( \gamma \) as functions of prestrain was implemented in ABAQUS via the user subroutine UMAT. A backward Euler algorithm is used for integrating the plastic strain increments, in which a trial stress is obtained by assuming that the strain increment is purely elastic deformation. Residuals of stress, back stress and yield function are defined, and are linearized using the first term of a Taylor series expansion about the trial states. From these expansions, the stresses, back stresses and other state variables are updated. Iteration continues until the residual of the yield function is less than the specified tolerance (typically the residual is required to
be less than $10^{-12}$ times the yield stress). Upon convergence, the consistent tangent modulus matrix is computed and used with a Newton–Raphson equilibrium iteration procedure.

5. Anisotropic hardening law determined from tension–compression tests

Special tension–compression tests were developed to probe the Bauschinger effect of sheet metal under in-plane deformation at large strains (Balakrishnan, 1999). These tests make use of a standard tensile specimen and special fixtures to apply a stabilizing load parallel to the thickness direction. Corrections are made to account for friction introduced by the stabilizing fixtures and for the small biaxial loading that is required. Tests were conducted starting with either a tensile prestrain followed by compressive loading after reversal (labeled “T-C”), or a compressive prestrain followed by tensile loading (labeled “C-T”). Prestrains of up to approximately 0.08 were achievable in compression, and the full range of strain in tension may be achieved. Fig. 8 shows Balakrishnan’s (1999) data for the C-T tests at several prestrains, along with nonlinear kinematic hardening curves obtained using Eqs. (8) and (9). (The T-C results, which are not shown in Fig. 8 for clarity, are very similar and the material laws derived from those tests are compared later).

The series of C-T or T-C reverse curves is used to obtain the necessary coefficients of the nonlinear kinematic/isotropic hardening model as follows. The location of the yield surface, or backstress, $\sigma_o$, is determined from the measured forward and reverse flow stresses at the reversal according to Eq. (11), and the size of the yield surface, $s$, is determined according to Eq. (10). The last term in Eq. (8), $d\sigma_o/d\epsilon$, is determined by reference to the reverse loading curve at an adjacent prestrain by approximating $d\sigma_o/d\epsilon$ by $\Delta\sigma_o/\Delta\epsilon$. With these terms in hand, Eqs. (8) and (9) can be used with the measured plastic moduli, $H_f$ and $H_r$, to determine the remaining parameters $C$ and $\gamma$. While this is a sufficient method in principle, exact values of $H_f$ and $H_r$ are not readily obtained without ambiguity. Therefore, while initial values of $H_f$ and $H_r$ are obtained from apparent slopes of the curves, these quantities are adjusted in order to obtain a satisfactory fit throughout the transient period following the reversal. The changes of $H_f$ and $H_r$ introduced by this procedure are within the accuracy of the direct fit of the slopes. The result of this analysis is presented at each prestrain in Table 3, and Fig. 9 shows the assumed piece-wise linear interpolation of parameters needed to complete construction of a general constitutive equation for any prestrain.

Fig. 8 shows that the derived material laws reproduce the major features of the reverse loading curves. At lower prestrains, typically less than 0.02, the reverse flow curves tend to rejoin the monotonic loading curve after a transient strain range. For such cases, the nonlinear kinematic hardening law fits the reverse flow curves well. As the prestrain increases, permanent softening is observed after the reversal. The nonlinear kinematic hardening law cannot reproduce this behavior well because the flow stress approaches the monotonic loading curve after the transient period.

The nonlinear kinematic hardening law can be considered to consist of a yield surface and a limiting surface (Chaboche, 1986), with the limiting surface expanding
Fig. 8. Nonlinear kinematic hardening model as fit to uniaxial compression–tension test data (Balakrishnan, 1999).
isotropically. To reproduce the permanent softening of the reverse flow, the reverse flow modulus $H_r$ must be set to be very small so that the approach rate of the reverse flow to the monotonic flow curve will be gradual. In this way, long-term softening can be accommodated for the reverse curve at the expense of reproducing the transient accurately. The experimental uniaxial tension–compression curve shows that the transient range decreases as the prestrain increases. Therefore, for the reverse flow at high prestrain, a reasonably close fit can be achieved by ignoring the early transient portion while closely approximating the long-term offset. While by no means perfect, particularly at the elastic–plastic transition, the standard non-linear kinematic hardening model fit in this way adequately reproduces the main features of reverse hardening, as shown in Fig. 8.

To complete the model needed for FEM simulation, the variation of $C$ and $\gamma$ must be known explicitly with prestrain. At zero prestrain, $H_r$ is taken to be the same as $H_f$. For intermediate prestrains, linear interpolation is used. For prestrains beyond the measured range, three assumptions were made: (a) a saturation of permanent softening is adopted; (b) the rate of reverse yield stress change with prestrain is taken to be the same as the plastic modulus of the monotonic loading curve, and (c) the value of $H_r$ levels off after the highest prestrain measurement available. The variations of parameters thus obtained are shown in Fig. 9. The differences between compression–tension tests (C-T) and tension–compression tests (T-C) are minor compared with the variation with prestrain. (Note that all such fitting is done considering only a single strain reversal, rather than multiple cycles.)

### 6. Anisotropic hardening law determined from reverse-bend tests

The richness of data available from a reverse-bend test is limited compared with that obtained from a uniaxial tension–compression test. At each instant, a range of strains is sampled, not only through the thickness (where strain varies approximately linearly with position) but also with location along the length of the specimen.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\epsilon_f$</th>
<th>$\alpha$ (Mpa)</th>
<th>$C$ (Mpa)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 6022-T4 (C-T)</td>
<td>0.0111</td>
<td>51.3</td>
<td>12896.0</td>
<td>260.3</td>
</tr>
<tr>
<td></td>
<td>0.0275</td>
<td>41.2</td>
<td>6224.4</td>
<td>175.3</td>
</tr>
<tr>
<td></td>
<td>0.0466</td>
<td>19.3</td>
<td>$-575.4$</td>
<td>$-2.3$</td>
</tr>
<tr>
<td>DQSK (C-T)</td>
<td>0.0132</td>
<td>18.8</td>
<td>1579.7</td>
<td>53.4</td>
</tr>
<tr>
<td></td>
<td>0.0405</td>
<td>13.8</td>
<td>2119.9</td>
<td>116.2</td>
</tr>
<tr>
<td></td>
<td>0.0514</td>
<td>18.4</td>
<td>$-453.9$</td>
<td>$-8.5$</td>
</tr>
<tr>
<td>HSLA (C-T)</td>
<td>0.0211</td>
<td>84.5</td>
<td>9618.0</td>
<td>135.3</td>
</tr>
<tr>
<td></td>
<td>0.0344</td>
<td>56.1</td>
<td>6369.4</td>
<td>134.0</td>
</tr>
<tr>
<td></td>
<td>0.0485</td>
<td>35.5</td>
<td>$-124.4$</td>
<td>$-0.7$</td>
</tr>
</tbody>
</table>
Fig. 9. Variation of nonlinear kinematic hardening parameters with prestrain. Points obtained by analysis of tension–compression test data (Balakrishnan, 1999). $H_0$ is the plastic modulus at the initial yield of the monotonic loading curve.
Fig. 10 shows the distribution of the equivalent plastic strain obtained at the end of the bending stage for the elements located along the longitudinal centerline at the surface of the sheet specimen. The maximum equivalent plastic strain is approximately 0.03 for HSLA and DQSK, and about 0.046 for 6022-T4; and the equivalent plastic strain decreases rapidly with the distance from the punch. A few elements close to the punch and near the sheet surface experience relatively large straining, while most of the elements achieve strains less than 0.01–0.02. The reverse-bend test is thus likely to reflect material hardening behavior within a limited strain range, typically less than 0.02. The resulting load–displacement curve depends on a range of strains at each instant, so it is not possible to discriminate accurately the effect of prestrain at the strain reversal.¹

With this limitation in mind, the reverse-bend test may be simulated using finite element procedures and models discussed in a previous section. For given values of $C$ and $\gamma$, the accuracy of the fit can be assessed by comparing the simulated and measured load–displacement curves. $C$ and $\gamma$ can be adjusted in a trial-and-error fashion (or using more systematic inverse procedures (Zhao, 1999; Zhao and Lee, 1999)) to obtain best-fit values. This process is facilitated because $C$ and $\gamma$ have

¹ It is possible in principle to discern large changes that occur in strain reversals at higher prestrains by changing the stroke amplitude of the bend/reverse-bend test and finding best-fit coefficients for each amplitude. However, at each amplitude, the fundamental problem remains that the data represents a range of prestrains up to a fixed maximum determined by the stroke amplitude.
qualitatively different effects on the uniaxial flow curve (Shen, 1999). Larger $\gamma$ (for fixed $C$) produces a higher reverse yield stress and shorter transient period. Larger $C$ (for fixed $\gamma$) decreases the reverse yield stress and results in a longer transient strain. Thus, by matching reverse yield stress and transient strain, or yield stress and initial slope, acceptable values of $C$ and $\gamma$ may be found.

A trial-and-error procedure was adopted in the current work, and replication of the measured load–displacement results was obtained with excellent agreement (Fig. 11). In the process of the fitting, however, it was noted that multiple choices of $C$ and $\gamma$ were capable of reproducing nearly identical fits to the experiments. Therefore, it appears that even constant values of the two parameters $C$ and $\gamma$ cannot be identified unambiguously, thus obviating the usefulness of inverse methods (Zhao, 1999; Zhao and Lee, 1999) or more complex material laws (for example, where $C$ and $\gamma$ depend on other variables).

7. Results

Fig. 11 presents the simulation results for the reverse-bend test making use of three constitutive equations. The curve labeled “C-T model” was generated using the constitutive equation from uniaxial compression–tension tests, as described previously. (The curves using T-C models are omitted from Fig. 11 for clarity. In all cases, the curves lie within two line-widths of the C-T curves presented.) The curves labeled by values of $C$ and $\gamma$ were generated using the constitutive equations obtained from the bend test results themselves; that is, they represent simulations with best-fit (but not unique, as discussed above) coefficients obtained from the same experiments. All equations fit the original bend region equally because there is no strain reversal. For the reverse leg, both anisotropic hardening constitutive equations are in close agreement with the experimental results, while simulations based on a standard isotropic hardening law diverge.

Results shown in Fig. 11 demonstrate that taking into account the Bauschinger effect is essential for accurate calculation of the stress distribution through a sheet specimen undergoing reverse-bend deformation. In sheet metal forming, bend and reverse-bend deformation occur whenever the sheet is drawn over a die radius or through a draw bead. Because springback is proportional to moment and inverse to modulus, an accurate stress distribution is essential in the prediction of the amount of springback associated with these types of forming process.

Fig. 12 examines the accuracy of the constitutive equations derived from bend tests (i.e. best-fit constant $C$ and $\gamma$) in reproducing the uniaxial tension–compression tests. The agreement is poor, even qualitatively, except at the smallest prestrains. For prestrains greater than 0.02, none of the key characteristics is reproduced properly: i.e. the yield stress, initial hardening slope, transient strain, and long-term offset from the monotonic hardening law. Thus, while the two anisotropic hardening constitutive equations are indistinguishable in terms of reverse-bend simulations, which average over a range of strains simultaneously, they diverge when stress and strain can be distinguished uniquely, as in the tension–compression test.
Fig. 11. Comparison of measurements to simulation results of the reverse-bend test using isotropic and anisotropic hardening models for (a) 6022-T4, (b) DQSK and (c) HSLA.
Fig. 12. Comparison of constitutive equations obtained from reverse-bend tests and tension–compression (T-C) and compression–tension (C-T) measurements for (a) 6022-T4, (b) DQSK and (c) HSLA.
8. Discussion

The reverse-bend test is an attractive alternative to tension–compression and reverse-shear tests for investigating the Bauschinger effect in sheet materials. It is easy to perform with standard equipment, the set-up time and specimen preparation are minimal, and the fixtures are readily constructed. However, as noted earlier, stress and strain are obtained only indirectly, and must be averaged over a range of values at each instant.

In attempts to overcome this difficulty (Jiang, 1997; Yoshida et al., 1998; Shen, 1999; Zhao, 1999; Zhao and Lee, 1999), a hardening model (such as a nonlinear kinematic hardening model) is typically assumed, and an optimization procedure is then used to obtain best-fit values of the material parameters. Because of the relatively meager data available for the inverse procedure, the material parameters are typically taken as constants. Although more parameters may be added and better fits possibly obtained, a large multiplicity of nearly equal inverse results may be obtained for a

![Diagram](image)

Fig. 13. Comparison of Modification I with the original C-T model. Notice that there are no changes to the reverse flow curves for prestrains greater than 0.015.
As shown in Figs. 8 and 9, the material parameters are not constants, thus the essential assumption needed for the inverse procedure is violated. Worse, even using constant parameters in the material hardening law allows several sets to give equally good fits to the test results.

Three artificial modifications of the C-T model for DQSK were constructed to assess the sensitivity of the reverse-bend test results to details of the constitutive material response. Each modification makes use of the nonlinear kinematic hardening model, but each employs an altered set of constants to represent alternate reverse loading responses. For Modification I, the yield surface size and strain hardening rate differ considerably only for prestrains less than 0.015 (Fig. 13a and b), while for Modification II the changes are apparent only for prestrains greater than 0.015 (Fig. 14a and b). For Modification III (Fig. 15a and b), the yield surface size is modified throughout the range of prestrains.

Fig. 16 shows the simulated force–displacement curves for reverse-bend tests carried out using the modified material models. (Note that the strain distributions at the
maximum displacement are presented in Fig. 10). For comparison, the experimental points (which are almost undistinguishable from the simulations using the C-T model; Fig. 11b) are also shown. Modification I gives reverse-bending loads which differ by 10% from the other models and experiments, while the others are virtually indistinguishable. Comparison of results from Modifications I and II confirms the supposition stated earlier that the reverse-bend test is mainly sensitive to the small-prestrain constitutive response, for strains less than 0.015. Modification II, while having markedly different stress–strain response for larger prestrains, shows no significant difference in simulated reverse bend tests compared with simulations based on the baseline material model (“C-T”). Thus, it should not be expected that the large-prestrain material response can be discerned by the reverse-bend test.

Modification III shows that even alternative constitutive laws which differ considerably at small prestrains may reproduce the measured reverse-bend test force–displacement curves with excellent agreement. This is particularly important in view of the qualitatively different physical behavior, which is apparent at various ranges of prestrain, Figs. 8 and 9. At small prestrains, the transient range is limited. After an
incremental strain of 0.01–0.02, the monotonic hardening curve is recovered. At larger prestrains, the transient region is extended and further hardening shows a permanent offset from the monotonic curve. As demonstrated by Modification III, the details of many such models could be chosen to reproduce the reverse-bend test load–displacement curves accurately. Therefore, while the reverse-bend test can detect, in an average sense, the existence of a Bauschinger effect, it cannot be used reliably in an inverse manner to obtain details of the correct constitutive material laws.

9. Conclusions

A nonlinear kinematic hardening model has been used to model the Bauschinger effect in three sheet alloys. The required hardening parameters were obtained by fitting load–displacement curves obtained from a reverse-bend test (Shen, 1999), and by re-analyzing tension–compression data appearing in the literature (Balakrishnan, 1999). Both of the models were used with finite element modeling to simulate the reverse-bending test.

The following conclusions were reached:

1. The constitutive models obtained from the tension–compression test and an inverse procedure applied to reverse-bend results reproduce reverse-bend test
measurements equally well. Thus, there is no inconsistency of behavior in the two deformation modes.

2. The constitutive models obtained by fitting to the reverse–bend test and tension–compression test show significant differences when evaluated in terms of their stress–strain response following a stress reversal.

3. Constitutive models obtained by an inverse procedure applied to the reverse-bend test are not unique, even under the assumption of independence of prestrain. Widely varying constitutive equations can produce identical load–displacement curves from reverse-bend tests.

4. The reverse-bend test is primarily sensitive to the material response for prestrains less than 0.015, and is nearly insensitive to the material behavior at higher prestrains.

5. Consideration of the Bauschinger effect is essential for accurate calculation of moments and springback.

6. The nonlinear kinematic hardening law cannot represent accurately the Bauschinger effect at larger prestrains, where a long-term offset of stress or strain is observed.

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