A practical two-surface plasticity model and its application to spring-back prediction

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Abstract

A practical two-surface plasticity model based on classical Dafalias/Popov and Krieg concepts was derived and implemented to incorporate yield anisotropy and three hardening effects for non-monotonous deformation paths: the Bauschinger effect, transient hardening and permanent softening. A simple-but-effective stress-update scheme avoiding overshooting was proposed and implemented. Constitutive parameters were fit to 5754-O aluminum alloy using uniaxial tension/compression data. Spring-back predictions using the resulting material model were compared with experiments and with single-surface material models which do not account for permanent softening. The two-surface model improved such predictions significantly as compared with single-surface models, while the differences between two-surface simulations and experiments were insignificant.

Keywords: Two-surface model; Permanent softening; Bauschinger effect; Transient behavior; Overshooting; Sheet metal forming

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1. Introduction

As a way to improve automotive fuel efficiency and environment impact, efforts are under way to replace conventional steels with aluminum, magnesium and high strength steel alloys. However, their often inferior formability and/or larger spring-back are technical obstacles to overcome. Spring-back is a critical factor in the quality of final products, making the designing of forming tools more difficult and expensive. One way to effectively overcome difficulties in proper tool design and process optimization for these advanced materials is to introduce accurate computational simulations, which require proper description of material deformation properties.

Since sheet spring-back is the elastic unloading response after complex, large-strain deformation paths such as those encountered in sheet metal forming operations (Wagoner et al., 2006), its accurate simulation requires a proper constitutive description incorporating complex behavior such as the (1) Bauschinger effect, (2) transient behavior (Laukonis and Wagoner, 1984; Chung and Wagoner, 1986; Doucet and Wagoner, 1987; Doucet and Wagoner, 1989; Kim et al., 2003) and (3) permanent softening (Geng and Wagoner, 2002; Geng et al., 2002; Chun et al., 2002a). As schematically illustrated in Fig. 1, the reverse loading curve following deformation shows a smaller magnitude of yield stress (Bauschinger effect). It then either rapidly converges to the original curve (transient behavior without permanent softening) or it eventually parallels the original curve (permanent softening).

Two main approaches have been used to describe the reverse loading behavior: one based on kinematic hardening (shifting of a single-yield surface) and the other involving multiple yield surfaces (Khan and Huang, 1995). The former model is based on linear kinematic hardening models proposed by Prager (1956) and Ziegler (1959) to describe the Bauschinger effect. To add the transient behavior, the linear model was modified to nonlinear models by Amstrong and Frederick (1966) and Chaboche (1986) by introducing

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Fig. 1. A schematic unloading curve after pre-(tensile) strain to illustrate the Bauschinger, transient and permanent softening behavior (the bottom halves of unloading curves are plotted by rotating 180° so that they are moved up to the top).
an additional term to Prager’s linear kinematic hardening model. The kinematic hardening model has been also combined (Hodge, 1957).

The Chaboche model was further generalized recently as a combined type model, utilizing a non-quadratic anisotropic yield function and the Ziegler kinematic hardening model (1959), based on the plastic work equivalence principle modified for kinematic hardening to properly define effective (or equivalent) quantities in stress and plastic strain rate (Chung et al., 2005). This modified Chaboche model only accounts for the Bauschinger and transient behavior, not the permanent softening (Kim et al., 2006).

The nonlinear kinematic hardening model has been also modified to incorporate the permanent softening as well as the Bauschinger effect and transient behavior (Geng and Wagoner, 2002; Geng et al., 2002; Chun et al., 2002a,b). These models have shown successful in predicting draw-bending spring-back.

As for multi-surface models, Mroz’s model involves multiple numbers of yield surfaces in which piecewise linear variation of hardening is defined (Mroz, 1967), while two-surface models proposed by Krieg (1975) and Dafalias and Popov (1976) define the continuous variation of hardening between two yield surfaces. In the original multi-surface model proposed by Mroz (1967), the predicted stress–strain curve is piecewise linear because of the constant plastic moduli (Khan and Huang, 1995). Therefore, an infinite number of yield surfaces are needed to predict a smooth nonlinear curve as proposed by Mroz and Niemunis (1987), while the two-surface model can represent realistic smooth hardening with the continuous plastic modulus. However, the two-surface model may lead to the discontinuous change of elasto-plastic stiffness after partial unloading and also may produce unrealistic behavior such as too strong ratcheting as addressed by Hashiguchi (1997) and Chaboche (1986). These problematic issues will be further discussed later in this work.

These multi/two-surface models have been formulated based on the isotropic Mises yield function and these can represent the Bauschinger and transient behavior as well as permanent softening. Although additional multi/two-surface models have been proposed later (Mroz et al., 1979; Hashiguchi, 1981, 1988; McDowell, 1985), their applications have been mainly limited to small deformation (Khoei and Jamali, 2005) especially for the one-dimensional cyclic behavior of solid structures.

In this work, a new two-surface model is developed by modifying the Dafalias/Popov and Krieg models to incorporate the Bauschinger effect, transient hardening and permanent softening. It can accommodate general anisotropic yield surfaces as well as the combined isotropic and kinematic hardening for both yield surfaces, in which the translation rule for the kinematic hardening is based on the Ziegler model and the total hardening is decomposed into the isotropic and kinematic parts by introducing a constant ratio as introduced by Krieg. In the two-surface model, hardening behavior is newly updated every time reloading occurs (from elastic to plastic) considering the gap between two yield surfaces, unlike single-surface modes in which hardening rules are prescribed initially at the onset of calculation.

The new two-surface model is implemented into the commercial finite element code, ABAQUS/Standard, using the user subroutine, UMAT (ABAQUS, 2004). A simple but effective stress-update scheme was also developed along with supplementary numerical algorithms to resolve the overshooting problem as well as to determine the reverse loading in plane stress cases. This model and numerical implantation are evaluated by simulating the draw-bend spring-back test and comparing results with standard constitutive models.
and experiments. The non-quadratic anisotropic yield function Yld2000-2d (Barlat et al., 2003) was used for this particular example case performed for AA5754-O aluminum alloy.

2. Theory

In the proposed two-surface model, two yield stress surfaces are defined as shown in Fig. 2: the (inner) loading surface and the (outer) bounding surface. These two surfaces have the same shape, defined as an anisotropic yield function. Both translate (kinematic hardening) and expand (isotropic hardening), respectively, in a general scheme. The current stress state is defined on the loading surface (at point a in Fig. 2) and a “corresponding stress” is defined on the bounding surface (at point A). The correspondence between points a and A is defined by the common yield surface normal directions. The gap between the current and corresponding stresses determines the hardening rate, which is initially steep but becomes smaller as the gap decreases. Since the two surfaces can make contact (without penetration) only at points sharing the same normal direction, contact is imposed to occur at the current stress and corresponding stress points in the model. The hardening behavior including the transient behavior is newly prescribed at every time reverse loading occurs, unlike the single-surface model.

Fig. 2. A schematic view of the two-surface model and two gap distances.
2.1. Flow formulation

The elasto-plastic formulation for the two-surface model is derived here by applying the normality rule in the materially embedded coordinate system. Consider the following yield function for the (inner) loading surface with the homogeneous function of degree $m$:

$$f(\sigma - \alpha) - \sigma_{\text{iso}}^{m} = 0.$$  \hspace{1cm} (1)

Here, $\sigma$ is the Cauchy stress and $\alpha$ is the back-stress, which defines the central position of the current yield stress surface (with $\alpha = 0$ initially). Also, $\sigma_{\text{iso}}$ is the effective stress, a measure of the size of the yield surface. For combined type hardening, the loading surface expands, while being translated by $\alpha$.

The plastic work increment, $dw$, becomes

$$dw = \sigma \cdot d\varepsilon^p = (\sigma - \alpha) \cdot d\varepsilon^p + \alpha \cdot d\varepsilon^p,$$  \hspace{1cm} (2)

where $d\varepsilon^p$ is the plastic strain increment. The effective (or equivalent) quantities are (denoted by superimposed bars) now defined considering the following modified plastic work equivalence relationship; i.e.,

$$dw_{\text{iso}} = (\sigma - \alpha) \cdot d\varepsilon^p = \sigma_{\text{iso}} d\varepsilon,$$  \hspace{1cm} (3)

where $d\varepsilon$ is the effective plastic strain increment. Note that $\sigma_{\text{iso}}$ is defined in Eq. (3) for the stress translated by the back-stress $\alpha$. Therefore, $\sigma_{\text{iso}}$ is obtained from the initial effective stress (which is relevant to the relationship, $\sigma d \varepsilon = \sigma_{\text{iso}} d\varepsilon$) by replacing $\sigma$ with $\sigma - \alpha$. Then, the effective plastic strain increment for the kinematic hardening in Eq. (3) becomes equivalent to the initial effective strain increment, therefore, the effective plastic strain increment surface is stationary.

As for the effective back-stress increment, $d\alpha$, the value is obtained from the initial effective stress by replacing $\sigma$ with $d\alpha$ as a conjugate quantity to $d\varepsilon$; i.e., $d\alpha \cdot d\varepsilon^p = d\alpha d\varepsilon$. The definitions of effective quantities for the stress, the conjugate plastic strain increment and the back-stress increment are defined for any anisotropic yield stress surface and they are first-order homogenous functions. In Eq. (2), another effective-like back stress quantity $\tilde{\alpha}$ can be defined by $\alpha \cdot d\varepsilon^p = \tilde{\alpha} d\varepsilon$. However, $\tilde{\alpha} \neq \tilde{\alpha}(= \int d\tilde{\alpha})$, in general. When monotonously proportional back-stress loading is assumed, $dw_{\alpha} = \alpha \cdot d\varepsilon^p = \tilde{\alpha} d\varepsilon = \tilde{\alpha} d\varepsilon$ so that $dw = \sigma_{\text{iso}} d\varepsilon + \tilde{\alpha} d\varepsilon = \tilde{\sigma} d\varepsilon$.

Differentiating Eq. (1) and applying the modified plastic work equivalence principle leads to

$$\frac{\partial f}{\partial (\sigma - \alpha)} d\sigma - \frac{\partial f}{\partial (\sigma - \alpha)} d\alpha - m\sigma_{\text{iso}}^{m-2} h_{\text{iso}}(\sigma - \alpha) \cdot d\varepsilon^p = 0,$$  \hspace{1cm} (4)

where $h_{\text{iso}}(\equiv \frac{d\sigma_{\text{iso}}}{d\varepsilon})$ is the slope of the isotropic hardening curve, $\sigma_{\text{iso}}$, as a function of the effective plastic strain $\varepsilon(\equiv \int d\varepsilon)$, while

$$d\alpha = dc_{1} v.$$  \hspace{1cm} (5)

In Eq. (5), $v$ represents the translational direction of the loading surface. The translation of the back stress is often assumed to be either the Prager model, $v \sim d\varepsilon^p$ or the Ziegler model, $v \sim (\sigma - \alpha)$. The Ziegler model ensures proportional plastic deformation for proportional loading (and vice versa), while the Prager model does not (Pourboh- rat et al., 1998). When the translational direction $v$ in Eq. (5) is defined in the stress
field including the case of the Ziegler model, the magnitude of the back-stress increment in Eq. (5) is obtained by substituting the back-stress increment into the yield function $f$. Then,
\[
f(d\mathbf{x}) = f(d\mathbf{c}_1 v) = \mathbf{d}c_1^m f(v) = \mathbf{d}c_1^m \bar{\sigma}_{iso}^m(v) = \mathbf{d}\bar{\sigma}^m, \tag{6}\]
where $\mathbf{d}\bar{\sigma} = f(\mathbf{d}\mathbf{x})^\top$, therefore,
\[
\mathbf{d}\mathbf{z} = \frac{\mathbf{d}\bar{\sigma}}{\bar{\sigma}_{iso}(v)} \mathbf{v} = \left(\frac{\mathbf{d}\bar{\sigma}}{\mathbf{d}\mathbf{\dot{e}}}\right) \frac{\mathbf{v}}{\bar{\sigma}_{iso}(v)}. \tag{7}\]

Note that the translational direction of the loading surface can be arbitrary but the Ziegler model type is utilized here: $\mathbf{v} \sim (\mathbf{\sigma} - \mathbf{\alpha})$.

Considering linear isotropic elasticity and the additive decomposition of the strain increment, the stress increment is
\[
\mathbf{d}\sigma = \mathbf{C} \cdot \mathbf{d}\mathbf{\dot{e}} = \mathbf{C} \cdot (\mathbf{d}\mathbf{\dot{e}} - \mathbf{d}\mathbf{\dot{e}}^p), \tag{8}\]
where $\mathbf{C}$ is the elastic modulus, while $\mathbf{d}\mathbf{\dot{e}}$ and $\mathbf{d}\mathbf{\dot{e}}^p$ are total and elastic strain increments, respectively. The plastic strain increment is defined by the normality rule as
\[
\mathbf{d}\mathbf{\dot{e}}^p = d\lambda \frac{\partial f}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} = d\lambda (m\bar{\sigma}_{iso}^{m-1}) \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} = \mathbf{d}\bar{\sigma} \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})}, \tag{9}\]
considering the modified plastic work equivalence principle in Eq. (3) and the following nature for the homogeneous function of degree one $\bar{\sigma}_{iso}: (\mathbf{\sigma} - \mathbf{\alpha}) \cdot \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} = \bar{\sigma}_{iso}$. Now, with Eqs. (7) and (8), Eq. (4) becomes
\[
\frac{\partial f}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} \cdot \mathbf{C} \cdot \mathbf{d}\mathbf{\dot{e}} - \frac{\partial f}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} \cdot \mathbf{C} \cdot \mathbf{d}\mathbf{\dot{e}}^p - \frac{\partial f}{\partial \bar{\sigma}_{iso}} \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} = \mathbf{d}\bar{\sigma} \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})}. \tag{10}\]
Substituting Eq. (9) into Eq. (10) leads to, after some manipulations,
\[
\mathbf{d}\mathbf{\dot{e}} = \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} \cdot \mathbf{C} \cdot \mathbf{d}\mathbf{\dot{e}} - \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} \cdot \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} \frac{\partial \bar{\sigma}_{iso}}{\partial (\mathbf{\sigma} - \mathbf{\alpha})} + \mathbf{h}_{iso}. \tag{11}\]
Therefore, for a given strain increment $\mathbf{d}\mathbf{\dot{e}}$ prescribed at every time increment, Eqs. (9) and (11) determine the plastic strain increment, while the stresses are updated by Eqs. (7) and (8) on the loading surface as Jaumann increments.

Similarly to the loading surface, the (outer) bounding surface is described as
\[
F(\mathbf{\Sigma} - \mathbf{A}) - \bar{\Sigma}_{iso}^m = 0, \tag{12}\]
where $\mathbf{\Sigma}$ and $\mathbf{A}$ are the stress and back-stress of the bounding surface, respectively. Also, $\bar{\Sigma}_{iso}$ represents the size of the bounding surface and it is pre-determined by the proper decomposition as described in Section 2.2. Since the bounding surface $F$ shares the same shape with the loading surface $f$ and the corresponding stress on the bounding surface $\mathbf{\Sigma}$ shares the same normal direction with the current stress $\mathbf{\sigma}$ at the loading surface, the following condition is satisfied:
\[
\mathbf{\Sigma} - \mathbf{A} = \frac{\bar{\Sigma}_{iso}}{\bar{\sigma}_{iso}} (\mathbf{\sigma} - \mathbf{\alpha}). \tag{13}\]
As for the back-stress evolution, the following condition is imposed in addition to Eq. (7):

$$dA - dA_1 = -d\mu(\Sigma - \sigma) \quad \text{or} \quad dA = dA_1 - d\mu(\Sigma - \sigma) = dA_1 - dA_2.$$  \hspace{1cm} (14)

The condition in Eq. (14) specifies that two surfaces relatively translate along the line between the current and corresponding stresses, which ensures that contact between two surfaces occur at current and corresponding stresses. For the second term of Eq. (14),

$$dA_2 = \frac{d\tilde{A}_2}{\tilde{\sigma}_{iso}} \cdot (\Sigma - \sigma),$$  \hspace{1cm} (15)

where $d\tilde{A}_2 = \tilde{\sigma}_{iso}(dA_2)$ and $\tilde{\sigma}_{iso} = \tilde{\sigma}_{iso}(\Sigma - \sigma)$ as similarly done in Eq. (7).

Note that in the original Dafalias/Popov model, the multiplier $d\mu$ in Eq. (14) is determined by the consistency condition for the bounding surface and has a simple form for the isotropic yield function like the Mises surface. Here, the multiplier describing the translation of the bounding surface is determined from the measured hardening curve, which is similar to the Krieg model. Note that, if the bounding surface does not move ($dA = 0$), the condition in Eq. (14) overrides the back-stress evolution law of the loading surface so that Eq. (7) is held for $\nu \sim \Sigma - \sigma$, as originally proposed in the Mroz model (Mroz, 1967; Khan and Huang, 1995). Also, for cases when the two surfaces move independently without the constraint in their relative motions shown in Eq. (14), Hashiguchi (1988) proposed a mathematical formulation for the “non-intersection condition” to avoid intersection between the two surfaces by modifying the flow rule for the generalized material with not only hardening but also softening behavior. Also note that the translation of the loading and bounding surfaces is effective only on the deviatoric plane in the general 3-D case since these are cylindrical surfaces (as defined in Eqs. (1) and (12)) aligned along the hydrostatic stress line for incompressible plasticity.

### 2.2. Hardening parameters

In the two-surface model, the expansion and translation of the two surfaces are parameterized such that the Bauschinger, transient and permanent softening behaviors are properly represented and are matched to the 1-D reference state (uniaxial tension). For the reference state, the stress relationship becomes

$$\tilde{\Sigma} = \tilde{\sigma} + \tilde{\delta} \quad \left( \frac{d\tilde{\sigma}}{d\tilde{e}} = \frac{d\tilde{\sigma}}{d\tilde{e}} + \frac{d\tilde{\delta}}{d\tilde{e}} \right),$$  \hspace{1cm} (16)

where $\tilde{\delta}$ is the gap between $\tilde{\Sigma}$ and $\tilde{\sigma}$, the stress states at the bounding and loading surfaces, respectively. Initially, the hardening of the bounding surface is prescribed (or assumed if insufficient data is available) for expansion, $\tilde{\Sigma}_{iso}(\tilde{e})$, and translation, $\tilde{A}(\tilde{e})$ (or $\tilde{A}_2(\tilde{e})$) with proper separation. Here, $\tilde{A}(\tilde{e})$ can account for the permanent softening. Then, $\tilde{\sigma}(\tilde{e})$ is obtained to account for the transient behavior every time reverse loading occurs, considering the prescribed (or measured) gap function $\tilde{\delta}(\tilde{e})$, which is dependent on $\tilde{\delta}_{in}$, the initial gap distance measured at the start of reverse loading. To properly account for the transient behavior and permanent softening, the gap function should be chosen by fitting the experimental data measured by tension/compression or compression/tension tests. Then, the separation of $\tilde{\sigma}$ into the expansion and translation, $\tilde{\Sigma}_{iso}(\tilde{e})$ and $\tilde{\sigma}(\tilde{e})$, is executed to properly account for the Bauschinger effect. As for the translation and expansion rules of the loading and bounding surfaces in the combined isotropic-kinematic hardening model, the following simple decomposition is utilized as
\[ d\bar{\sigma} = (1 - m_1) d\bar{\sigma} + m_1 d\bar{\sigma} = d\bar{\sigma}_{\text{iso}} + d\bar{\epsilon}, \]
\[ d\bar{\Sigma} = (1 - m_b) d\bar{\Sigma} + m_b d\bar{\Sigma} = d\bar{\Sigma}_{\text{iso}} + d\bar{\lambda}, \]

(17)

where \( m_1 \) and \( m_b \) are the ratios of the kinematic hardening for the loading and bounding surfaces, respectively. Note that the parameters, \( m_1 \) and \( m_b \), are the functions of the accumulative plastic strain in general, however, constant values are assumed here for simplicity. Therefore, if the ratios become zero, the two surfaces expand without translation (isotropic hardening), whereas if the ratios become unity, the surfaces translate without expansion (kinematic hardening).

The scalar parameter \( \bar{\delta} \) to measure the gap between the current stress at the loading surface and the corresponding stress at the bounding surface is defined here as:

\[ \bar{\delta} = \sigma_{\text{iso}}(\Sigma - \sigma) = f^{\bar{\lambda}}(\Sigma - \sigma), \]

(18)

which is the effective stress value obtained by replacing \( \sigma - \sigma \) with \( \Sigma - \sigma \). Since the gap is effective only on the deviatoric plane for incompressible plasticity, if the gap is used to decide the expansion and translation of the loading surface, the gap defined in Eq. (18) is appropriate for that purpose. Eq. (18) is also useful for an anisotropic material, since a single \( \bar{\delta}(\bar{\epsilon}) \) relationship can be used for various loading directions, especially for proportional loading. As a simple example, consider the case for which the bounding surface is fixed without a size change, while the loading surface undergoes pure isotropic hardening, then a single \( \bar{\delta}(\bar{\epsilon}) \) relationship is shared for all (proportional) loading directions, even for anisotropic cases.

2.3. Non-quadratic anisotropic yield function: Yld2000-2d

In order to describe the anisotropic yield stress surface, the yield stress function Yld2000-2d (Barlat et al., 2003) for the plane stress state is incorporated for both the inner and bounding surfaces. The yield function has eight anisotropic coefficients so that it can accommodate eight mechanical measurements such as \( \sigma_0, \sigma_{45}, \sigma_{90}, r_0, r_{45}, r_{90} \), which are simple tension yield stresses and \( r \)-values (width-to-thickness plastic strain ratio in simple tension) along the rolling direction, 45° off and transverse directions as well as the yield stress \( \sigma_b \) and \( \sigma_{b}(=\sigma_{xx}/\sigma_{yy}) \) under the balanced biaxial tension \( (\sigma_{xx} = \sigma_{yy}) \) condition, respectively. Note that the convexity of the yield surface is well proven for this particular yield surface (Barlat et al., 2003).

The anisotropic yield function is defined as

\[ f = \phi' + \phi'', \]

(19)

where

\[ \phi' = |X'_1 - X'_2|^m, \quad \phi'' = |2X''_1 + X''_1|^m + |2X''_1 + X''_2|^m. \]

(20)

Here, \( f \) is the sum of two isotropic functions, which are symmetric with respect to \( X_1 \) and \( X_2 \). The resulting yield surface is convex for \( m \geq 1.0 \). In Eq. (20), \( X_1 \) and \( X_2 \) (\( X_1 \geq X_2 \)) are

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1 If \( \bar{\delta} \) is defined as \( \sigma_{\text{iso}}(\Sigma - \sigma) \), then \( \sigma_{\text{iso}}(\Sigma - \sigma) = \sigma_{\text{iso}}(-\Sigma) - \sigma_{\text{iso}}(\sigma) \) so on, which is improper since the gap becomes insensitive to the sign of stress quantities, while if it is defined as \( \|\Sigma_{\text{deviatric}} - \sigma_{\text{deviatric}}\| \), it is not so effective for anisotropy since the definition does not account for the directional difference.
3. Numerical implementation

3.1. Stress update procedure

For the numerical formulation, the incremental deformation theory (Chung and Richmond, 1993) was applied to the elasto-plastic formulation based on the materially embedded coordinate system. Under this scheme, the strain increments in the flow formulation become discrete strain increments, which are the true (or logarithmic) strain increments along with two hardening curves, \( \sigma_{\text{iso}}(\bar{\varepsilon}) \) and \( \tilde{\sigma}(\bar{\varepsilon}) \), which are prescribed in advance once reloading occurs as discussed in Section 2.2. After \( \Delta \bar{\varepsilon} \) is obtained as a solution of Eq. (23), updated stress and back-stress are obtained from \( \bar{\sigma}_{\text{iso}}(\bar{\varepsilon} + \Delta \bar{\varepsilon}) \) and \( \tilde{\sigma}(\bar{\varepsilon} + \Delta \bar{\varepsilon}) \), respectively, while the new configuration of the bounding surface is also updated at the end of each step considering \( \Sigma_{\text{iso}}(\bar{\varepsilon}) \), \( A_2(\bar{\varepsilon}) \) and \( \tilde{\sigma}(\bar{\varepsilon}) \) for \( \Delta \bar{\varepsilon} \). Note that the two hardening curves of the loading surface, \( \bar{\sigma}_{\text{iso}}(\bar{\varepsilon}) \) and \( \tilde{\sigma}(\bar{\varepsilon}) \), are newly updated every time reloading occurs, considering the new initial gap distance \( \delta_{\text{in}} \).

The predictor–corrector scheme based on the Newton–Raphson method was used to solve \( \Delta \bar{\varepsilon} \) in Eq. (23) for the loading surface; i.e.,

\[
\bar{f}(\sigma_0 - \mathbf{a}_0 + \Delta \sigma(\Delta \bar{\varepsilon}) - \Delta \mathbf{a}(\Delta \bar{\varepsilon})) = \bar{\sigma}_{\text{iso}}(\bar{\varepsilon}_0 + \Delta \bar{\varepsilon}),
\]

(23)

In Eq. (22), there are eight anisotropic coefficients: \( \alpha_1 \sim \alpha_8 \). The yield function reduces to the isotropic expression when all the independent coefficients \( \alpha_k \) (\( k = 1–8 \)) become 1.0. When only seven coefficients are needed to account for seven measured values for example \( \sigma_0, \sigma_{45}, \sigma_{90}, r_0, r_{45}, r_{90} \) and \( r_b \), it may be assumed that \( x_1 = x_6 \) (therefore, \( L_{12}'' = L_{21}'' \)) or \( x_4 = x_5 \) (therefore \( L_{11}'' = L_{22}'' \)). A detailed procedure to obtain yield parameters from experiments is discussed elsewhere (Barlat et al., 2003).
After Eqs. (25) and (26) are considered, Eq. (24) is a nonlinear equation to solve for the time step. Therefore, 

\[ \sigma_{n+1}^T = \sigma_n + C \cdot \Delta \varepsilon \quad \text{and} \quad 0 \leq \beta \leq 1. \]  

(26)

After Eqs. (25) and (26) are considered, Eq. (24) is a nonlinear equation to solve for \( \Delta \varepsilon \), when \( \Delta \varepsilon \) is given. Then, linearization of Eq. (24) leads to

\[ \delta(\Delta \varepsilon)^k + 1 = -\frac{\phi^k}{\delta \Delta \varepsilon}, \]

(27)

for the \( k \)th iteration and

\[ \frac{\partial \sigma}{\partial \Delta \varepsilon} = \frac{\partial \sigma}{\partial \sigma_{n+1}} \frac{\partial \sigma_{n+1}}{\partial \Delta \varepsilon} + \frac{\partial \sigma}{\partial \sigma_{n+1}} \frac{\partial \sigma_{n+1}}{\partial \Delta \varepsilon} + \frac{\partial \sigma}{\partial \sigma_{n+1}} \frac{\partial \sigma_{n+1}}{\partial \Delta \varepsilon}, \]

(28)

where

\[ \frac{\partial \sigma_{n+1}}{\partial \Delta \varepsilon} = -C \cdot \frac{\partial \sigma_{n+1}}{\partial \sigma_{n+1}} \quad \text{and} \quad \frac{\partial \sigma_{n+1}}{\partial \Delta \varepsilon} = \frac{\partial \sigma_{n+1}}{\partial \sigma_{n+1}} \frac{\sigma_{n+1} - \sigma_{n+1}}{\sigma_{n+1}}, \]

(29)

as well as

\[ \frac{\partial \sigma_{n+1}}{\partial \sigma_{n+1}} = -1. \]

(30)

Note that in deriving Eq. (29), the higher-order terms caused by the variation of stresses with respect to the variation of the effective strain increment have been ignored for simplicity.

After \( \tilde{\sigma}_{n+1} \) (therefore, along with \( \sigma_{n+1} \) and \( \sigma_{n+1} \)) is obtained for the loading surface, the current stress on the bounding surface \( \Sigma_{n+1} \) and its center \( \Lambda_{n+1} \) (or \( \Delta A_2 \)) are obtained from the following two conditions:

\[ \Lambda_{n+1} = \Lambda_n + \Delta \Sigma_{n+1} - \Delta A_2 (\Delta \tilde{\sigma}_{n+1}) \frac{\Sigma_{n+1} - \sigma_{n+1}}{\sigma_{iso}(\Sigma_{n+1} - \sigma_{n+1})}, \]

(31)

\[ \Sigma_{n+1} - \Lambda_{n+1} = \frac{\Sigma_{iso}(\Delta \tilde{\sigma}_{n+1})}{\sigma_{iso}(\Delta \tilde{\sigma}_{n+1})}(\sigma_{n+1} - \sigma_{n+1}), \]

(32)

which are two simultaneous equations for the stress on the bounding surface and the center of the surface: \( \Sigma_{n+1} \) and \( \Lambda_{n+1} \). Therefore, adding Eqs. (31) and (32), the following nonlinear equation is obtained for the unknown quantity \( \Sigma_{n+1} \):

\[ \Phi = \Sigma_{n+1} - \frac{\Sigma_{iso}(\Delta \tilde{\sigma}_{n+1})}{\sigma_{iso}(\Delta \tilde{\sigma}_{n+1})}(\sigma_{n+1} - \sigma_{n+1}) - \Lambda_n - \Delta \Sigma_{n+1} + \Delta A_2 (\Delta \tilde{\sigma}_{n+1}) \frac{\Sigma_{n+1} - \sigma_{n+1}}{\sigma_{iso}(\Sigma_{n+1} - \sigma_{n+1})} = 0. \]

(33)
Linearizing Eq. (33) for the Newton–Raphson method provides, for the $k$th iteration,
\[
\delta \Sigma_{n+1}^k = - \frac{\Phi^k}{\left( \frac{\partial \Phi}{\partial \Sigma_{n+1}} \right)^k},
\]
where
\[
\frac{\partial \Phi}{\partial \Sigma_{n+1}} = I + \frac{\Delta A_2(\Delta \bar{\varepsilon}_{n+1})}{\sigma_{iso}(\Sigma_{n+1} - \sigma_{n+1})} - \Delta A_2(\Delta \bar{\varepsilon}_{n+1}) \frac{(\Sigma_{n+1} - \sigma_{n+1}) \otimes \bar{\sigma}_{iso}}{\partial \Sigma_{n+1}}.
\]
Here, $I$ is the second-order identity tensor. After solving $\Sigma_{n+1}$ from Eq. (34), $A_{n+1}$ is obtained from Eq. (32).

### 3.2. Problematic issues of two-surface model

The characterization of hardening curves for the bounding and loading surfaces from the simple tension test (or a reference state) is a complex task. The hardening behavior of the loading surface is newly updated in the two-surface model every time reloading occurs considering the initial gap distance $\delta_{in}$. When the separation of the isotropic hardening and the kinematic hardening in the loading surface is performed, caution should be paid to gaps $\delta$ and $\bar{\delta}$ shown in Fig. 2, especially when the loading path is not proportional. The gap $\delta$ is the distance between the current stress on the loading surface and the corresponding stress on the bounding surface (marked a and A in Fig. 2), while the gap $\bar{\delta}$ is the distance between the corresponding stresses (marked b and B in Fig. 2) aligned with the line connecting two centers of the loading and bounding surfaces. Since the two surfaces relatively translate (along the line between a and A) and expand, the two surfaces meet at points a and A if the kinematic hardening is dominant in the inner surface, while the two surfaces meet at points b and B if expansion by the isotropic hardening is dominant in the inner surface. Note that premature contact at b and B should be avoided in the two-surface model with the proper separation of the isotropic and kinematic hardening in order not to penetrate the bounding surface. One possible way to avoid this problem is to decompose the total hardening into kinematic and isotropic hardening of the inner surface so as for its isotropic rate to be less than that of the outer surface. Here, the effective value $\bar{\varepsilon}$ is defined as
\[
\bar{\varepsilon} = \bar{\sigma}_{iso}(\Sigma - \sigma_b) = f_1^\frac{1}{n}(\Sigma - \sigma_b),
\]
which becomes
\[
\bar{\varepsilon} = \bar{\xi} = \Sigma_{iso} - \bar{\sigma}_{iso} - f_1^\frac{1}{n}(x - A) = \bar{\sigma}_{iso} - \bar{\sigma}_{iso}(x - A).
\]
In Eq. (37), the first two terms are values defining the current sizes of the loading and bounding surfaces. The gap $\delta$ is larger than the gap $\bar{\varepsilon}$ in general for non-proportional loading.

Another commonly addressed unrealistic transient behavior is known as “overshoot-ing” (Khan and Huang, 1995). This problem occurs when the material is (almost) elastically unloaded before it is reloaded to the original stress state (the state before the elastic unloading) for plastic deformation as schematically illustrated for the 1-D case in Fig. 3. For reloading in such a circumstance, realistic flow stress follows the previous hardening behavior in a continuous manner, therefore the newly updated hardening behavior based on the new initial gap $\delta_{in}$ falsely overshoots the hardening. Using the previous hardening
behavior without updating is one way to resolve the problem as suggested by Tseng and Lee (1983) and Dafalias (1986) for the 1-D case. In the plane stress case, considering the nature of bounding surface concept in which the hardening behavior is updated whenever reverse loading occurs for plastic deformation, it is efficient to properly define the reverse loading criterion such that the new initial gap distance $\delta_{\text{in}}$ is updated only when the reverse loading criterion is satisfied. Fig. 4 shows the reverse loading criterion introduced here, in which $\theta_d$ is the angle between two subsequent stresses on the loading surface, while $\theta_r$ is a prescribed reference angle for reverse loading: the reverse loading condition that 

$$0 \leq \theta_r \leq \theta_d \leq \pi$$

where

$$\theta_d = \cos^{-1} \left( \frac{\sigma - \alpha}{\sigma - \alpha_{\text{old}}} \cdot \frac{\sigma - \alpha}{\sigma - \alpha_{\text{old}}} \right) = \cos^{-1}(d_{\text{new}} \cdot d_{\text{old}}).$$

The hardening update which involves the new gap function and reverse loading condition is performed considering the following linear combination associated with new and previous initial gaps and the parameters $\gamma_a$ and $\gamma_b$, which are the functions of the angle $\theta_a$:

$$\delta = (1 - \gamma_a) \cdot \delta_{\text{old}}^{\text{in}}, \quad \bar{\varepsilon}_i = (1 - \gamma_b)\bar{\varepsilon}_{\text{old}}$$

$$\delta = \delta_{\text{new}}^{\text{in}}, \quad \bar{\varepsilon}_i = (1 - \gamma_b)\bar{\varepsilon}_{\text{old}}, \quad 0 \leq \gamma_{a,b} \leq 1,$$

$$\delta = \delta_{\text{new}}^{\text{in}}, \quad \bar{\varepsilon}_i = 0, \quad \gamma_{a,b} > 1.$$

Here, $\delta_{\text{old}}^{\text{in}}$ and $\delta_{\text{new}}^{\text{in}}$ are the initial gap distances for the previous and the current loading curves, respectively, while $\bar{\varepsilon}_i$ and $\bar{\varepsilon}_{\text{old}}$ are the initial plastic strain for the reversed hardening curve and the strain at the load reversal from the previous loading curve, respectively. The parameters are supposed to be experimentally determined, however, $\gamma_a = \gamma_b = \frac{\theta_d}{\theta_r}$ is assumed here for simplicity. For instance, when the reverse loading criterion is $\theta_r = \frac{\pi}{2}$, previous hardening is used with the equivalent strain at the load reversal for $\theta_d = 0$ (corresponding to $\gamma_a = \gamma_b = 0$), while new hardening is initialized for $\theta_d = \frac{\pi}{2}$ (corresponding to $\gamma_a = \gamma_b = 1$) or larger. The numerical algorithm for the two-surface model described in this session is summarized in Table 1.
4. Numerical examples

4.1. Random cyclic loading

Even though the main purpose of this work is to apply the two-surface model for the simulation of sheet forming particularly for spring-back, the model was applied first for a random cyclic loading under a uniform uniaxial stress condition in order to verify the implementation of formulations. The hardening function proposed by Dafalias and Popov (1976) was utilized:

\[
\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \chi(\bar{\sigma}_{\text{in}})\left(\frac{\bar{\delta}}{\bar{\sigma}_{\text{in}} - \bar{\delta}}\right).
\] (40)

Here, \(\chi\) is a parametric function of \(\bar{\sigma}_{\text{in}}\) controlling the steepness of the stress–strain hardening curve, which can be assumed a linear function of \(\bar{\sigma}_{\text{in}}\) or a more complex one (Usami et al., 2000). Eq. (40) is the differential equation for \(\bar{\delta}\) whose solution provides \(\bar{\delta} = \bar{\delta}(\bar{\varepsilon})\) where \(\bar{\varepsilon}\) is the plastic strain whose value is re-initialized for each reverse loading, while \(\bar{\varepsilon}\) is the accumulated equivalent plastic strain (as \(\bar{\varepsilon} = \sum \bar{\varepsilon}_i\)). The numerical exercise was performed with the function of the form

\[
\chi = \frac{a}{1 + b(\bar{\sigma}_{\text{in}}/\bar{\sigma}_t)^m},
\] (41)

where \(\bar{\sigma}_t\) is a reference stress to make the term \(\bar{\sigma}_{\text{in}}/\bar{\sigma}_t\) dimensionless. Material parameters from the literature (Dafalias and Popov, 1976) was utilized: \(a = 164E_0\), \(b = 46.0\), \(m = 3\), \(\bar{\sigma}_t = 156\) MPa, and \(E_0 = 4.42\times10^3\) MPa (the tangent modulus of bounding curve). For simplicity, only kinematic hardening was assumed for both the loading and bounding surfaces: \(m_l = m_b = 1\) in Eq. (17). With kinematic hardening in the bounding surface, permanent softening exists in this example.

![Fig. 4. Reverse loading criterion for the two-surface model.](image)
Table 1
Numerical algorithm for the two-surface model

A.1. Elastic predictor
- Evaluate the trial elastic stress at the loading surface for a given discrete strain increment
\[ \sigma_{n+1}^{(k=1)} = \sigma_n + C \cdot \Delta e, \quad \mathbf{a}_{n+1}^{(k=1)} = \mathbf{a}_n, \quad \Sigma_{n+1}^{(k=1)} = \Sigma_n, \quad A_{n+1}^{(k=1)} = A_n \]
and \[ \tilde{e}_{n+1}^{(k=1)} = \tilde{e}_{n+1}^{(k=1)} \]
- Check the yield condition for the loading surface
  - If \[ f^2(\sigma_{n+1}^{(k=1)} - \sigma_n^{(k=1)}) - \tilde{\sigma}_{\text{iso}}^{(k=1)}(\tilde{e}_{n+1}^{(k=1)}) < \text{Tol} \], then set \( (*)_{n+1} = (*)^T \) Exit.
  - Else Go to step A.2.

A.2. Plastic corrector for the loading surface
- Initialize
\[ k = 1, \quad \Delta \tilde{e} = 0, \quad \sigma_{n+1}^{(k=1)} = \sigma_n, \quad \mathbf{a}_{n+1}^{(k=1)} = \mathbf{a}_n, \quad \Sigma_{n+1}^{(k=1)} = \Sigma_n, \quad A_{n+1}^{(k=1)} = A_n \]
and \[ \tilde{e}_{n+1}^{(k=1)} = \tilde{e}_{n+1}^{(k=1)} \]
- Check the reverse loading criterion by Eq. (38)
  - If the reverse loading criterion is satisfied, calculate the new initial gap distance \( \tilde{\sigma}_{\text{in}} \) and construct all the hardening laws for the stress and the back-stress at the loading and bounding surfaces.
  - If the reverse loading criterion is not satisfied, do not update the hardening laws
- Check the yield condition
\[ \phi_{n+1}^{(k)} = f^2(\sigma_{n+1} - \mathbf{a}_{n+1}) - \tilde{\sigma}_{\text{iso}}(\tilde{e}_{n+1}) \]
- If \[ \phi_{n+1}^{(k)} < \text{Tol} \], then set \( (*)_{n+1} = (*)^{(k)}_{n+1} \) Go to A.3.
- Else Go to next
  - Evaluate the addition to the equivalent plastic strain increment
\[ \delta(\Delta \tilde{e}) = - \frac{\partial \Phi}{\partial \Delta \tilde{e}} \]
where
\[ \frac{\partial \Phi}{\partial \Delta \tilde{e}} = \frac{\partial \Phi}{\partial \sigma_{n+1}} \frac{\partial \sigma_{n+1}}{\partial \Delta \tilde{e}} + \frac{\partial \Phi}{\partial \mathbf{a}_{n+1}} \frac{\partial \mathbf{a}_{n+1}}{\partial \Delta \tilde{e}} + \frac{\partial \Phi}{\partial \tilde{\sigma}_{\text{iso}}} \frac{\partial \tilde{\sigma}_{\text{iso}}}{\partial \Delta \tilde{e}} \]
- Re-evaluate the discrete increments of the effective strain and stresses
- Update variables
\[ \Delta \tilde{e}_{n+1}^{(k+1)} = \Delta \tilde{e}_{n+1}^{(k)} + \delta(\Delta \tilde{e}) \]
\[ \sigma_{n+1}^{(k+1)} = \sigma_{n+1}^{(k)} + \Delta \sigma_{n+1}(\Delta \tilde{e}_{n+1}^{(k+1)}) \]
\[ \mathbf{a}_{n+1}^{(k+1)} = \mathbf{a}_{n+1}^{(k)} + \Delta \mathbf{a}_{n+1}(\Delta \tilde{e}_{n+1}^{(k+1)}) \]
Set \( k = k + 1 \) and continue iteration.

A.3. Update the bounding surface (and \( \Sigma_{n+1}, A_{n+1} \))
- Known values: \( \Delta \tilde{e}_{n+1}, \sigma_{n+1}, \mathbf{a}_{n+1} \)
- Update the hardening curves \( \Sigma_{n+1} \) and \( A_{n+1} \)
- Obtain \( \Sigma_{n+1} \) from the Newton–Raphson method
\[ \delta \Sigma_{n+1} = - \frac{\partial \Phi}{\partial \Sigma_{n+1}} \]
Fig. 5a shows a random uni-axial strain history for a random displacement history boundary condition considered here and the resulting calculated stress–strain relationship is shown in Fig. 5b. The calculated curve reproduces the Bauschinger and transient behavior. The result confirms that the stress update procedure is properly implemented. In the strain history between B and C, the overshooting was suppressed by the algorithm.

4.2. Spring-back in 2-D draw bending test

In order to apply the two-surface model to spring-back prediction, a benchmark 2-D draw bending test (NUMISHEET, 1993) was performed for the aluminum alloy AA5754-O sheet with 1.51 mm thickness. The hardening behavior during loading and unloading was measured utilizing the continuous tension/compression test, which was performed with a designed device preventing buckling under compression (Lou et al., 2007; Boger et al., 2005; Lee et al., 2005a). The tension/compression curves for several pre-strains are plotted in Fig. 6. The compressive (reverse) hardening behavior after pre-straining in tension is shown as a function of accumulated absolute strain. Note that the tension/compression test results in Fig. 6 are plotted by rotating the original figures by 180° with respect to the origin and then the bottom halves were moved up to the top. The reverse curves show early re-yielding (Bauschinger effect) and rapid change of work hardening rate (transient behavior). Also, the permanent gap between the monotonic curve in tension (without pre-strain) and the reverse curve shows a “permanent” softening phenomenon, which increases as the amount of pre-strain increases.

In order to represent the Bauschinger, transient and permanent softening behavior, the two-surface model was applied. For the bounding surface, purely kinematic hardening (without isotropic hardening) was assumed, while the bounding stress–strain curve is represented by linear hardening with a constant slope:

\[
\Sigma = d + \epsilon e. \tag{42}
\]

Here, for the model material AA5754-O, constant values of \( d = 233 \) MPa and \( e = 250 \) MPa were utilized.

As for the loading surface, combined isotropic-kinematic hardening was assumed with the following gap function:

\[
\delta = a(\delta_m) + b(\delta_m) \exp(-c(\delta_m)\epsilon^l). \tag{43}
\]
All the three material parameters depend on the initial gap stress $\bar{\sigma}_{\text{in}}$ in Eq. (42). From the experimental data, the material parameters $a$, $b$ and $c$ in Eq. (43) are obtained by considering gap distances between bounding and loading stress. The dependence of these three parameters on the initial gap stress was assumed piecewise linear as shown in Fig. 7. Considering the bounding surface hardening in Eq. (43) and the gap in Eq. (43), the isotropic-kinematic hardening used constant $m$ values as discussed in Eq. (17): $m_l = 0.4$, $m_b = 1.0$. Using the measured material parameters, the tension/compression test data were re-calculated using the two-surface model. Comparison of the calculated and measured hardening behavior shown in Fig. 8 confirms that the two-surface model represents the hardening data well including the Bauschinger and transient behavior as well as the permanent softening.

For comparison purpose, the hardening behaviors of the (pure) kinematic, the (pure) isotropic hardenings as well as the modified Chaboche model are included in Fig. 8. The modified Chaboche model is a combined isotropic-kinematic hardening model, which
accounts for the Bauschinger and transient behavior but not the permanent softening behavior, while the (pure) isotropic hardening does not account for the Bauschinger, transient nor permanent softening. The formulation and material characterization procedure for the modified Chaboche model is documented elsewhere (Lee et al., 2005b). Even though both the two surface model and modified Chaboche model are properly parameterized for experiments, their unloading performance with respect to the permanent softening is different as shown in Fig. 8 so that their prediction capability for spring-back becomes different, as will be discussed later with respect to Fig. 11.

Isotropic elastic properties were assumed: 70 GPa Young’s modulus, 0.33 Poisson’s ratio. As for the anisotropic yield function Yld2000-2d, seven anisotropic coefficients were obtained (assuming \( a_3 = a_6 \) by measuring three uniaxial yield stresses, \( \sigma_0, \sigma_{45} \) and \( \sigma_{90} \), and three \( r \)-values (the width-to-thickness plastic strain increment ratio in uni-axial tension), \( r_0, r_{45} \) and \( r_{90} \) along the rolling (0°), transverse (90°) and in-between (45°) directions as well as the balanced biaxial yield stress (\( \sigma_b \)). The uniaxial tensile properties were measured using the uniaxial tensile test and the balanced biaxial yield stress was measured using the hydraulic bulge test. The detailed characterization procedure is documented elsewhere (Lee et al., 2005a) and the results are shown in Table 2.

A schematic view of tools and dimensions for the 2-D draw bending test is shown in Fig. 9a. The dimensions of die gap, punch and die sizes were slightly modified from original benchmark values to accommodate the thickness of the test material. The initial dimension of the blank sheet was 300 mm (length, rolling direction) \( \times \) 35 mm (width). The limiting punch stroke was 70 mm and the blank holder force (BHF) was 2.5 kN. Considering the geometric symmetry of the process, only half of the blank was simulated using the commercial finite element code ABAQUS/Standard (ABAQUS, 2004) with user-defined material subroutine UMAT. The 4-node three-dimensional rigid body element, R3D4 was used for tools and the reduced four-node shell element, S4R with nine integration points through thickness was employed for the blank. The friction coefficient between the tools and the sheet blank was chosen to be 0.1, as recommended by the NUMISHEET benchmark committee (1996).
Fig. 7. Material parameters for the two-surface model.
Fig. 8. Comparison of stress–strain curves between measured and calculated curves: (a) pre-strain = 0.023, (b) pre-strain = 0.05, and (c) pre-strain = 0.078.
The spring-back of the 2-D draw bending test was simulated using the two-surface model as well as three single-surface models: the (pure) isotropic hardening model, the (pure) kinematic hardening model and the modified Chaboche model. The reverse loading behavior generated by the four models is compared in Fig. 8. Fig. 10 shows the comparison of final shapes after spring-back between experiment and four simulation results. Also, spring-back angles and flange heights are quantitatively compared between experiments and simulations as listed in Table 3. Here, the spring-back angle and the flange height are defined in Fig. 9b. The simulation result with the current two-surface model shows the best prediction over the other hardening models. The results of the isotropic hardening and modified Chaboche models were very similar each other, both

<table>
<thead>
<tr>
<th>$m$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
<th>$x_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.879</td>
<td>1.136</td>
<td>0.952</td>
<td>1.048</td>
<td>1.009</td>
<td>0.952</td>
<td>1.034</td>
<td>1.237</td>
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</table>

Fig. 9. (a) A schematic view of tools and dimensions for the 2-D draw bending test and (b) typical spring-back profile after forming.
over-predicting the spring-back, while the kinematic hardening under-predicted the spring-back. Various reference angles for $\theta_t$, larger than zero, were tested but the results were the same because loading is almost proportional for this particular test case.

As for the simulation results of the four models, the moment–curvature curves of a sheet material which undergoes bending, unbending and reverse bending with a slight tensile force was considered as illustrated in Fig. 11. Because material elements in punch and die corners in the 2-D draw bending test experience unloading after bending and stretching up to a given curvature and a tensile force, the curvature recovery $\kappa_u$ alludes to the magnitude of spring-back at those corners. Thus, the difference of this curvature recovery is not so significant for all hardening models. However, material elements at the sidewall undergo bending with stretching up to a given die curvature and then unbending/reverse bending under a tension force until they are straightened before unloading. Therefore, the curvature recovery $\kappa_f$ refers to the magnitude of spring-back at the sidewall. The schematic and simulated moment–curvature curves of a particular element on the side wall region for the four hardening models are illustrated in Fig. 11 a and b, respectively. The schematic results were obtained from simplified pure bending calculations, while the calculated ones were obtained from the finite element simulation of the draw-bending test. Note that the value of curvature at the initiation of spring-back is not exactly zero in the simulation results, due to the gap between the die cavity and the punch.

Table 3
Quantitative comparison between experiments and simulations

<table>
<thead>
<tr>
<th>Models</th>
<th>$\theta_{\text{meas.}}$ (°)</th>
<th>$\theta_{\text{simul.}}$ (°)</th>
<th>error ($\Delta \theta$)$^a$</th>
<th>% Error</th>
<th>$\Delta z_{\text{meas.}}$ (mm)</th>
<th>$\Delta z_{\text{simul.}}$ (mm)</th>
<th>% Error$^b$</th>
</tr>
</thead>
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<tr>
<td>Two-surface</td>
<td>15.20</td>
<td>16.46</td>
<td>1.26</td>
<td>8.29</td>
<td>53.05</td>
<td>51.71</td>
<td>2.52</td>
</tr>
<tr>
<td>Isotropic</td>
<td>23.84</td>
<td>8.63</td>
<td>56.78</td>
<td>43.34</td>
<td>18.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kinematic</td>
<td>6.73</td>
<td>8.48</td>
<td>55.74</td>
<td>62.19</td>
<td>17.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modified Chaboche</td>
<td>23.10</td>
<td>7.98</td>
<td>51.94</td>
<td>45.35</td>
<td>14.51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ $\frac{|\theta_{\text{simul.}} - \theta_{\text{meas.}}|}{\theta_{\text{meas.}}} \times 100$ (%).

$^b$ $\frac{\Delta z_{\text{simul.}} - \Delta z_{\text{meas.}}}{\Delta z_{\text{meas.}}} \times 100$ (%).
Without the Bauschinger effect and transient nor permanent softening, the isotropic hardening model showed the maximum $\kappa_f$, while the kinematic hardening model showed the minimum recovery because of the effect of the Bauschinger effect without expansion of yield surface. The two-surface model showed the intermediate recovery because of the effect of the Bauschinger, transient and permanent softening behavior. In the modified Chaboche model, the moment–curvature during unloading rapidly converged to that of the isotropic hardening model so that $\kappa_f$ was similar to that of the isotropic hardening model, with the minimum effect of the Bauschinger and transient behavior. Therefore, the main cause of the different simulation results is the permanent softening behavior, which was properly accounted for only by the two-surface model. Even though there are other ways to properly represent the softening behavior in material models (Geng and Wagoner, 2002), the 2-D draw simulation result demonstrated that the two-surface model adequately accounts for the softening behavior and that this is important for reverse loading behavior including the spring-back.

Fig. 11. Moment–curvature curves in (a) schematic and (b) simulated results: (1) the (pure) isotropic hardening model (2) the modified Chaboche model (3) the two-surface model (4) the (pure) kinematic hardening model.
5. Conclusions

1. A practical two-surface plasticity model has been developed based on classical Dafalias/Popov and Krieg concepts. Initial yield anisotropy is effectively incorporated, as are complex hardening effects for non-monotonous loading: Bauschinger effect, transient yield and permanent softening.
2. The new model was numerically implemented in ABAQUS Standard/UMAT utilizing a simple stress-update scheme that avoids overshooting.
3. A numerical procedure for fitting all required hardening parameters based on sheet tension/compression was introduced and used to characterize 5754-O aluminum alloy. Tension/compression experiments were closely reproduced by the model.
4. Draw-bend spring-back simulated with the two-surface model matched experimental results within the precision of the measurement. Simulations based on single-surface models not incorporating permanent softening were significantly less accurate.
5. Permanent softening affects draw-bend spring-back significantly.
6. Accurate sheet spring-back prediction requires taking into account complex hardening for non-monotonous paths, particularly permanent softening.

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