42 Sheet Springback

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42.1 Introduction

As used in this chapter, *springback* refers to the usually undesirable and often deleterious shape change that occurs following a press-forming operation when the forming loads are removed from the work piece. It is driven by internal stress patterns that are equilibrated with external loads at the end of the forming operation. Springback is particularly important for long, slender structures such as sheet-formed parts because small changes in stress pattern across the thickness translate into significant changes of the radius of curvature, which in turn translate into large displacements normal to the thickness.

The large displacements inherent in sheet springback lead to many problems downstream of the forming operation, including difficulties in subsequent forming operations, final assembly (particularly welding), appearance, and quality. These problems have a significant economic impact. A recent estimate (Wenner 2001) for the U.S. automotive industry alone put the cost of springback at more than $50 million per year, and a similar estimate has been made for U.S. Air Force applications (Semiatin 2001). In spite of this, there has been relatively little research done on springback related to typical sheet forming practice.

There are two basic approaches to avoiding the drawbacks inherent in springback: 1) minimizing the springback displacements, usually by increasing sheet tension, and 2) compensating for predicted or measured springback by tooling or process changes. (A third possible approach, designing subsequent operations and final products for springback tolerance, doesn't address the root problem and will not be considered here.) For operations where the sheet tension cannot be controlled (e.g. in pure bending), compensation is the only alternative. Fortunately, pure bending is the simplest case to model and can usually be predicted and compensated for, although differences in material properties and process variables lead to unavoidable scatter.

For typical press-forming operations, such as those encountered in the automotive industry, control of sheet tension via drawbeads and other tooling features has been the historical mainstay of successful forming practice. Tool design optimization has usually been accomplished using rules-of-thumb for the initial design and then trial-and-error tooling changes until the final desired part shape is obtained. However, as product cycle times continue to decrease, the need to understand, predict, and compensate for springback at the die design stage has become increasingly important. While simulation techniques for sheet forming operations have been nearly universally adopted, the current challenge is applying these techniques in a reliable, predictive way to incorporate springback prediction.

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At the time of this writing, the challenges facing control of springback in automotive-type forming operations concentrate into several areas: 1) reliable and robust numerical techniques for predicting springback following a sheet forming operation of a complex part, 2) material properties and mechanics formulations capturing the essential material behavior of the work piece during and after forming, and 3) design methods that make use of springback simulations of presumed accuracy to guide changes in tooling without the need for experimental intervention (i.e. trial and error). Because of space limitations, design methods making use of springback simulations will not be reviewed in this chapter, but the reader is referred to recent papers that have been submitted for publication in the open literature (Gan and Wagoner 2003a, Gan et al. 2003b).

This chapter is aimed at providing a brief introduction of the principal literature of springback, both analytical and experimental. The extensive references should be sufficient to enable the reader to trace the known knowledge in the area. An even briefer review of some recent developments is presented based on the draw-bend test results obtained by the author and co-workers. This chapter is thus organized into three principal parts, as follows:

1. Review of Simulation Literature
2. Review of Experimental Literature
3. Draw-Bend Springback

42.2 Review of Simulation Literature

A variety of analytical and simulation techniques have been applied to springback. These generally may be classified as analytical (usually closed form) or numerical (typically finite element) techniques.

Analytical Solutions Analytical solutions for springback of plane stress, pure bending were presented for elastic-perfectly plastic material response (Gardiner 1957), and were extended to plane-strain pure bending (Queener and De Angelis 1968), and initially curved cases (Shaffer and House 1955). Later solutions were for plane-strain bending with superimposed tension and with a wider range arbitrary R/t (radius of curvature-to-thickness ratio) (Baba and Tozawa 1964, Ingrarson 1975, Ueda et al. 1981, Yu and Johnson 1982, Yuen 1990, Tozawa 1990, Wang et al. 1993, El-Domiaty et al. 1996, Zhang and Hu 1997). Narrow plane-stress strips have been analyzed for bending as well (Chan and Wang 1999). Reviews of pre-1950's developments have been presented (Nadai 1950, Phillips 1956), as have more recent analytical springback methods and approaches (Huang and Gerdeen 1994, Yu and Zhang 1996).

Analytical methods other than finite element modeling have been applied to springback in die forming, often with limitations to pure bending, as appropriate for U-bending (Sudo et al. 1974), flanging (Wang 1984), sidewall curl (Thompson and Ellen 1995), repeated bending unbending (Chu 1986, Kuwabara et al. 1996a) or stamping with deformable tools (Zhang and Lin 1997). These analyses rely on radii of curvature during the forming operation established by geometrical considerations of the dies, but do not usually solve explicitly for such shapes consistent with the material mechanics and contact friction with the tools. With superimposed tension, these methods may be applied to plane-strain draw die forming (Yoshida 1965, Duncan and Bird 1978, Wenner 1983) with gentle tooling curvatures. Such methods have also been applied to stretch bending of channels (Ueda et al. 1981, Mickalich and Wenner 1988) or draw bending of top-hat sections (Zhang and Lee 1995). Empirical rules have also been used to treat springback (Levy 1984).

Finite Element Modeling With the rapid increase in computation power, finite element methods (FEM) for analyzing and predicting springback have become more attractive. Recent benchmark tests and accompanying papers (Makinouchi et al. 1993, Shen et al. 1995, Lee et al. 1996, Gelin and Picart 1999, Yang et al. 2002) illustrate the state of the art in predicting springback with FEM. In particular, the 1993 benchmark (Makinouchi et al. 1993), Figure 42.1a, represents a flanged channel forming operation that was simulated by many commercial and special-purpose programs with widely varying results, Figure 42.1b. This result was surprising in view of the reliability developed in forming simulations by that period, however, it should be noted that the experimental scatter, as approximated by the error bars on Figure 42.1b, generally exceeded the scatter of the simulations.

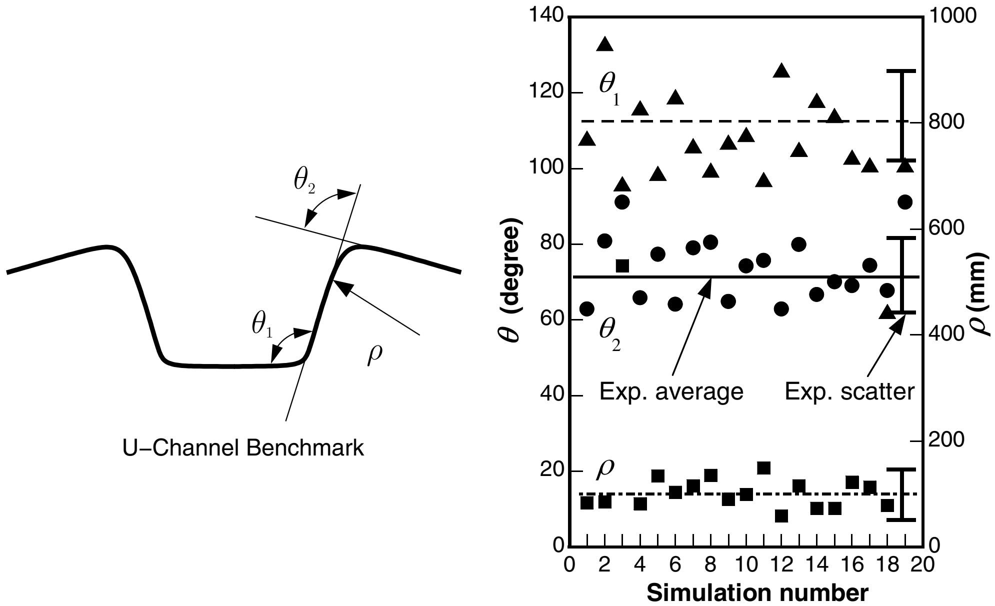

Figure 42.1: U-channel springback benchmark of NUMISHEET '93 (Makinouchi et al. 1993). Each point on the plot represents a simulation result while the error bars represent the approximate scatter of the experimental results.

FEM simulations of springback are much more sensitive to numerical tolerances than are forming simulations (Mattiasson et al. 1995, Wagoner et al. 1997, Lee and Yang 1998, Li et al. 1999b), including effects of element type (Li et al. 1999a), integration scheme (He and Wagoner 1996, Focellese et al. 1998, Lee and Yang 1998, Li et al. 1999a,b, Narasimhan and Lovell 1999), and unloading scheme (Tang 1987, Yuen 1990, Li et al. 1999b). Because of the inherent numerical sensitivity, implicit schemes for both loading and unloading i.e. implicit/implicit) have been popular (Wagoner et al. 1997, Hu and Du 1999, Li et al. 1999a,b, Geng and Wagoner 2000), as well as attempts to link dynamic explicit simulations of forming operations (upon which many commercial programs are based) to static implicit simulations of springback (Mattiasson et al. 1995, He and Wagoner 1996, Park et al. 1999, Valente and Traversa 1999). Nearly every possible approach to simulating springback has been recently reported as successful, including explicit/explicit (Montmayeur and Staub 1999), static explicit/static explicit (Kazama et al. 1999), and even one-step methods (Abdelsalam et al. 1999). Hybrid approaches have been developed for post-processing of forming FEM results for springback (Pourboghrat and Chu 1995) and for optimizing die design by iterating with analysis (Karafillis and Boyce 1992a,b, Karafillis and Boyce 1996, Ghouati et al. 1998).

Springback is also sensitive to a range of material and process parameters, such as strain hardening (Wenner 1983, Mickalich and Wenner 1988, Zhang and Lee 1995, Han and Park 1999), evolution of elastic properties (Chakhari and Jalinier 1984, Morestin and Boivin 1996), elastic and plastic anisotropy (Chakhari and Jalinier 1984) and the presence of a Bauschinger effect (Baba and Tozawa 1964, Tozawa 1990, Pourboghrat and Chu 1995, Zhang and Lee 1995, Kuwabara et al. 1996a, Tang 1996, Focellese et al. 1998, Kuwabara et al. 1999).

42.3 Review of the Experimental Literature

Corrections for springback are essential during die design in order to obtain specified final shapes. Correction curves based on empirical information or simple theories for small-springback in pure bending cases have been available for many decades (Schroeder 1943, Sachs 1951, Levy 1984) based on the basic assumptions of engineering beam bending. Many analytical solutions have been derived, and reviews have appeared both for the early work (Nadai 1950, Phillips 1956), and more recent contributions (Huang and Gerdeen 1994, Yu and Zhang 1996).

While the literature dealing with analysis of springback of metal sheets is extensive, carefully-controlled experiments of springback under realistic forming conditions (i.e. involving bending and unbending simultaneous with imposed tension and sliding over the tooling) are less common. Many experiments have been carried out under pure-bending (i.e. with minimal tension) conditions of various kinds: cylindrical tooling (Yu and Johnson 1983, Yuen 1990, Sanchez et al. 1996), U-bending/channel bending (Sudo et al. 1974, Chakhari and Jalinier 1984, Hino et al. 1999), V bending (Chakhari and Jalinier 1984, Zhang and Hu 1997, Hino et al. 1999), and flanging (Wang 1984). Such experiments show springback increasing with R/t (tool radius/sheet thickness), but such results have little application to situations where significant sheet tension is present, because sheet tension dominates other process variables in determining springback.

Stretch-bend tests (Ueda et al. 1981, Kuwabara et al. 1996b, Hino et al. 1999) allow careful control of sheet tension during bending, but do not typically exhibit bending and unbending, nor large sliding over the tooling common in press forming operations. These tests do show, however, the pervasive effect of sheet tension on springback, especially for sheet tensile stresses in the range of the material yield stress.

Actual forming operations, such as a two-dimensional idealization of draw/stretch bending of a flanged channel (also called a top-hat section) are used to assess practical springback. This geometry is the most-studied of springback cases (Ayres 1984, Davies 1984, Hayashi 1984, Umehara 1990, Schmoeckel and Beth 1993, Pourboghrat and Chu 1995, Bayraktar and Altintas 1996) (including benchmark testing and simulations (Makinouchi et al. 1993)) because of its practical importance, and because of the obvious, measurable presence of sidewall curl. However, in such operations, the sheet tension is determined only indirectly, either from the blank holder force, or from a drawbead simulation (Nine 1978, Nine 1982, Wang 1982, Nine 1983), both of which depend on the coefficient of friction, which is usually known only approximately. Because of the dominant role of sheet tension in springback, lack of control or direct measurement of this quantity is a serious drawback for scientific use of such experiments in verifying simulation techniques. Such operations have common features, including reduced springback and curl for increased blank holder forces (especially when sheet tension approaches the yield stress). Curl has been shown to disappear as R/t approaches 2 for a variety of steels (Davies 1984, Hayashi 1984, Umehara 1990). For smaller R/t, the curl may reverse direction.

The exceptional work is by Liu (1988) for flanged channels, and Kuwabara et al. (1996b) for draw bending. Liu devised a special channel flanging experiment in which the restraining force was applied directly by a controlled hydraulic cylinder. In this way, sidewall curl and springback could be assessed in terms of known restraining forces for several materials. Kuwabara et al. used a draw-bend apparatus that appears similar to the one in the current work, although dimensions and specifications were not provided (Kuwabara et al. 1996b). Reported springback angles decreased with increasing R/t and the usual dependence on sheet tension was observed. The current draw/bend test (Wenzloff et al. 1992, Haruff et al. 1993, Vallance and Matlock 1992), conceived for friction testing, is similar in concept to Liu's experiment, with four significant differences of the current device:

1. much longer draw distances are attainable, with correspondingly increased precision of shape measurement,
2. there is no bend at the bottom of the channel to complicate interpretation,
3. there is only a single tool, thus avoiding changing clearances and geometries during the punch stroke, and
4. the material hardening curves were measured and fit in some detail.

The sensitivity of springback to a Bauschinger effect, particularly for bend/unbend operations (as is the case for draw/bend tests and flanged channel forming) has been noted in several analyses (Baba and Tozawa 1964, Yoshida 1965, Tozawa 1990, Pourboghrat and Chu 1995, Zhang and Lee 1995, Tang SC, 1996, Focellese et al. 1998, Kuwabara et al. 1999). However, until recently, the magnitude of the Bauschinger effect in sheet metal tension-compression (Kuwabara et al. 1995c, Balakrishnan 1999) or bend/reverse bend (Jiang 1997, Shen 1999) had seldom been measured because of the instability of metal sheets subjected to in-plane compression.

A variety of manufacturing techniques has been developed for dealing with springback. The simplest involves designing tools to over-bend the sheet to compensate for springback.

However, this is not applicable to complex curved parts and, without additional control, this approach can lead to large variations in practice as material properties and thickness vary. Other approaches include the use of deformable tools (Zhang and Lin 1997, Zhang et al. 1997), adjustments to small die radii and clearances (Hayashi 1984, Umehara 1990), through-thickness deformation (Chou and Hung 1999), variable or stepped blank holder force control (Tozawa 1990, Hishida and Wagoner, 1993, Schmoeckel and Beth 1993, Sunseri et al. 1994, Sunseri et al. 1996, Han and Park 1999), multiple forming steps (Ayres 1984, Nagai 1987, Tozawa 1990), or reconfigurable tooling (Kutt et al. 1999). Each of these methods can benefit from accurate knowledge of springback from measurement and analysis.

42.4 Draw-Bend Springback

As outlined above, the existing springback literature has either a) limited relevance to standard sheet forming because of different forming conditions, or b) high scatter arising from imprecise control of sheet tension in industrial configurations. The draw-bend test (Vallance and Matlock 1992, Wenzloff et al. 1992, Haruff et al. 1993) avoids the shortcomings of many of the previous test configurations. As shown schematically in Figure 42.2a, the test reproduces closely the geometry of drawing a sheet over a die radius (such as into a die cavity). The strip is bent around a fixed radius and then straightened as it is drawn for large distances under significant sheet tension. The test not only reproduces industrial sheet forming conditions, it may be conducted with precise closed-loop control of the sheet tension. The large specimen geometry allows measurement of springback angles to within 0.3°, corresponding to an outer fiber strain of 0.00001 (Wagoner et al. 1997).

Draw-Bend Results The author and coworkers have used the draw-bend test as a basis for springback experiments and corresponding simulations (Li and Wagoner 1998, Li et al. 1999a,b, Geng and Wagoner 2000, Wagoner et al. 2000a,b, Carden et al. 2002, Li et al. 2002, Geng and Wagoner 2002). The simulations make use of either research finite element programs SHEET-S (Wagoner et al. 1989, Saran et al. 1991c, Wang and Wagoner 1991a,b, Keum et al. 1992) and SHEET-3 (Germain et al. 1989, Keum et al. 1989, Wagoner et al. 1990, Kim and Wagoner 1991, Saran and Wagoner 1991a,b, Wagoner and Zhou 1992, Wagoner and Zhou 1993, Zhou and Wagoner 1994, Zhou and Wagoner 1995), or commercial program ABAQUS (ABAQUS User Manual, version 6.2, version 6.2.1.). Comparison of measurements with simulations, Figure 42.2b, reveals the complexity of the springback behavior and its simulation under these conditions. (Note that the *normalized back force* in Figure 42.2b represents the controlled sheet tension expressed as a fraction of the sheet tension required to yield the strip specimen.)

Numerical Sensitivity Initial 2-D simulations, using standard numerical parameters and material laws for sheet forming simulations (Figure 42.2b, Curve 1), showed dramatic discrepancies with the experiments. A 2D sensitivity analysis (Li and Wagoner 1998, Li et al. 1999a, b) of the simulation results in terms of numerical parameters was undertaken to verify the consistency of the simulations themselves. On this basis and considering later 3D simulations, the following guidelines were reached for simulating sheet springback:

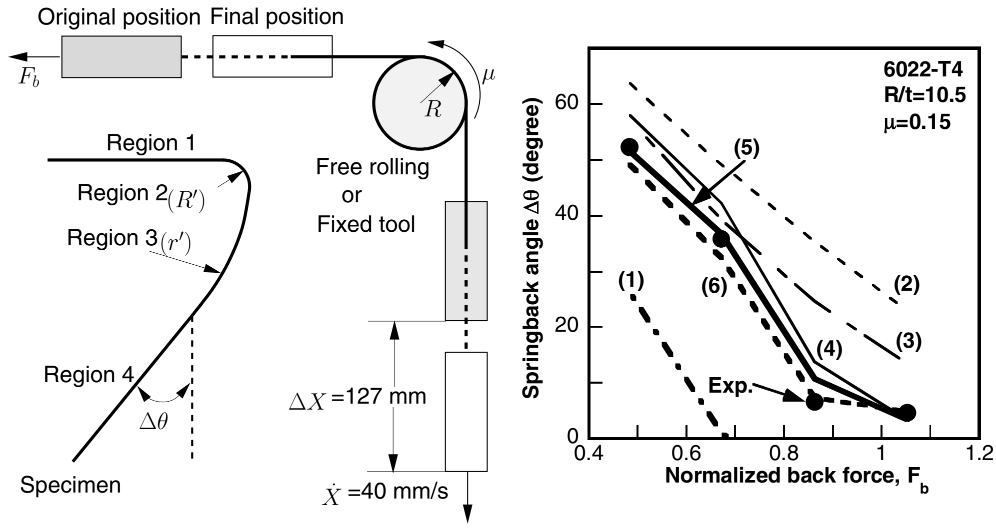

Figure 42.2: Improvement of draw-bend springback prediction by choice of numerical procedure and better material understanding (Li and Wagoner 1998, Li et al. 1999a,b, Geng and Wagoner 2000, Wagoner et al. 2000a,b, Carden et al. 2002, Li et al. 2002, Geng and Wagoner 2002). $<\sigma>$ represents the standard error of fit for each simulation relative to measurements. Unless otherwise noted, von Mises yield and isotropic hardening were employed. Notation:
(1) Plane-stress simulation, 5 integration points (IP), 600 elements along length (El), $<\sigma>= 26^\circ$
(2) Plane-strain simulation, 51 IP, 600 El (remaining results use 51 IP, 600 El), $<\sigma>= 19^\circ$
(3) Plane-stress simulation, $<\sigma>= 11^\circ$
(4) 3-D shell simulation (w/persistent anticlastic curvature), $<\sigma>= 5.7^\circ$
(5), (6) 3-D shell/Geng-Wagoner anisotropic hardening law, Barlat '96 yield function, $<\sigma>= 2.3^\circ$ and 1.2°, respectively.

- Up to 51 integration points through the sheet thickness are required (compared with 3 to 5 normally used for sheet forming simulation)
- Fine meshes, typically 4 times finer (or less than 5 degrees of turning angle per element) as compared with sheet-forming meshes
- Convergence must be insured to very tight tolerances, typically to one part in 10,000
- Because of the numerical sensitivity, it is essential to perform sensitivity analyses
- Implicit time integration appears to be essential in view of the numerical sensitivity to convergence tolerances (He and Wagoner 1996)
- 3-D elements are required because of persistent anticlastic curvature
- In some cases, it is necessary to consider elastic-plastic springback
- For R/t ratios less than 5, shell elements are not sufficient and solid elements are required

It should be noted that these requirements increase the computational requirements for forming/springback simulation considerably in comparison to forming alone, not only in raw computation time, but in the reliability of the algorithms employed. The time for simulation of forming and springback can increase by a factor of 100 times that required for the forming step alone.

Anticlastic Curvature Once the numerical parameters and procedures had been satisfactorily established, the 2D simulation results were still hundreds of percent in error for those cases where the sheet tensile stresses approached the material yield stress, although for other back forces far removed from this value the agreement was much better. (Compare Curve 3 with the experimental results for a normalized back force near 0.9. and 0.5, for example.)

3-D shell simulations and re-measurement of the specimens revealed the presence of persistent anticlastic (secondary) curvature, the role of which in nominally 2D springback testing was not previously appreciated (Carden et al. 2002, Li et al. 2002). The sudden depression of springback angle for sheet tension approaching yield was shown to be a result of this effect.

Role of Yield Function The magnitude of anticlastic curvature, and therefore the springback angle, were shown to depend intimately on the shape and anisotropy of the yield function. Maintaining an isotropic hardening rule but changing the yield function form yielded the following standard errors of fit (not shown on Figure 42.2b for clarity reasons):

Hill quadratic yield (Hill 1948, Hill 1950):	11.3°
Barlat YLD89 (Barlat and Lian 1989):	10.2°
Von Mises:	5.7°
Barlat YLD96 (Barlat 1997):	2.0°

As discussed more fully elsewhere (Geng and Wagoner 2002), the springback simulation is intimately tied to the form of the yield function in two respects: 1) the lateral (TD) yield stress controls the persistence of anticlastic curvature, with a lower yield enhancing the permanent secondary curvature that remains after springback (thus increasing the moment of inertia and decreasing springback drastically), and 2) the strain ratio arising from the normal to the yield function control the under-load anticlastic curvature in the same way that Poisson's ratio controls anticlastic curvature in an elastic beam.

It is because of these competing effects that the Von Mises yield function works fortuitously while the Hill quadratic (Hill 1948, Hill 1950) and Barlat YLD 89 (Barlat and Lian 1989) do not. These latter yield functions, which are nearly equivalent, capture either the r-value variation with direction well but not the yield stress variation, or vice-versa. Figure 42.3 shows that only the Barlat YLD 96 (Barlat 1997) yield function matches both yield stress and plastic strain ratio variations accurately, and thus reproduces all of the aspects of material behavior necessary to understand and predict draw-bend springback.

3-D simulations with appropriate numerical tolerances and standard material laws continued to show significant variance with the experiments: 5.7° standard deviation vs. 1° experimental scatter. Two aspects of the material constitutive law were found responsible for the errors: the shape of the initial yield function and the assumption of isotropic hardening (Geng and Wagoner 2000, Wagoner et al. 2000b). When these aspects were investigated with

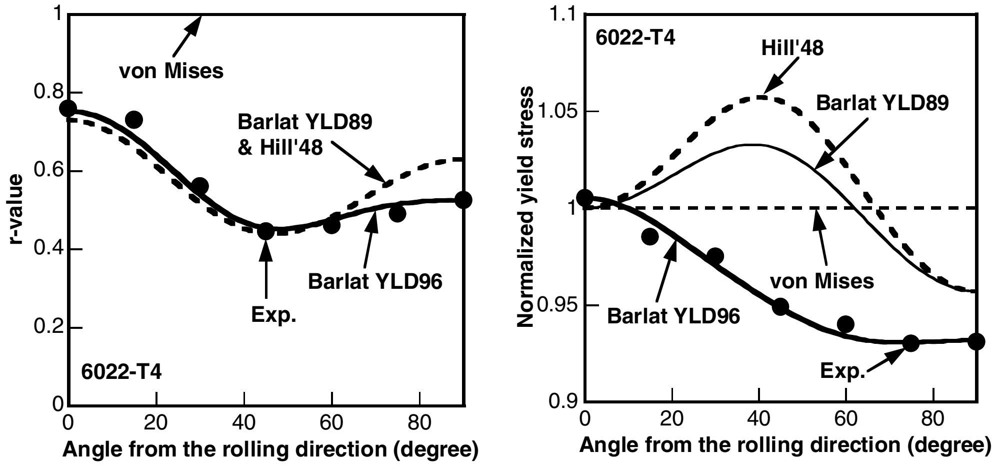

Figure 42.3: Plastic anisotropy of aluminum alloy (Barlat et al. 1997,Shen 1999, Geng and Wagoner 2002).

independent experiments and taken into account by algorithmic development and implementation, the simulations and measurements agreed within the experimental scatter (Figure 42.2b, Curves 5 and 6).

Role of Bauschinger Effect A second aspect of the material law was found to influence draw-bend springback results significantly. Because nearly every element of material through the sheet thickness in Region 3 of the specimen, the dominant area, undergoes stress and strain reversal, the presence of a Bauschinger effect must be considered. In order to investigate the Bauschinger effect for the 6022-T4 alloy tested, a stabilized tensile test, Figure 42.4a, was constructed (Balakrishnan 1999, Geng and Wagoner 2002) using principles presented in the literature (Kuwabara et al. 1995a, b). Typical results of these tests are shown in Figure 42.4b.

Bauschinger effects of the type shown in Figure 42.4b are not reproduced well by continuum isotropic hardening models, nor by most forms of kinematic hardening (Armstrong and Frederick 1966, Krieg 1975, Dafalias and Popov 1976, Chaboche et al. 1979, Ristinmaa 1995, Jiang and Kurath 1996). Based on Armstrong-Frederick-type hardening rules (Chaboche et al., 1979, Chaboche and Rousselier 1983, Chaboche 1987, Chaboche and Nouailhas 1989, Chaboche 1991, Ohno and Wang 1991, Ohno and Wang 1993a, b, Jiang and Sehitoglu 1995a, b, Khan and Huang 1995), Geng and Wagoner (2000) introduced a variation based on a two-surface plasticity model which reproduces the strain-hardening behavior adequately. To model the permanent softening shown on reverse loading at larger prestrains, the bounding surface translates and expands according to a mixed hardening rule (Hodge 1957, Crisfield 1991). This hardening law has been implemented for plane-stress thin-shell elements in conjunction with three anisotropic yield criteria: Hill'48 (Hill 1948, Hill 1950, Mellor and Parmar 1978), Barlat's three-parameter yield function (Barlat Yld89) (Barlat and Lian 1989) and Barlat's Yld96 (Barlat 1997).

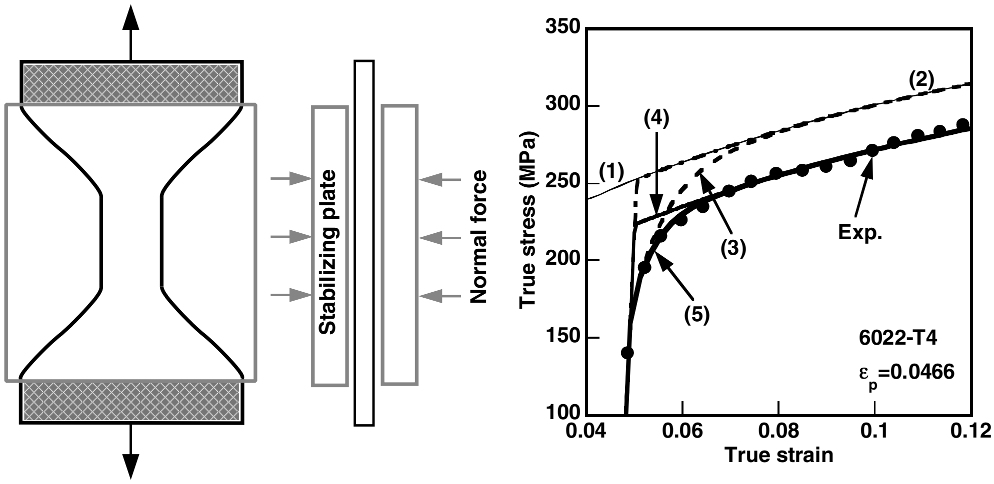

Figure 42.4: Measurement of the large strain Bauschinger effect: (a) schematic of sheet tension compression test and (b) comparison of reverse flow curves with different plastic hardening models. Notation: (1) Monotonic flow curve, (2) isotropic hardening, (3) Nonlinear kinematic hardening (Chaboche), (4) Mixed hardening, (5) Modified hardening (Geng-Wagoner).

When this hardening law was introduced along with the Barlat YLD96 yield function, the springback (and anticlastic curvature) predictions matched the experiments within the scatter of the experiments, 1.2° (Curve 6, Figure 42.2b).

The results presented above illustrate the complex and sensitive dependence of springback phenomena on the detailed plasticity law, including anisotropy, complex hardening, and Bauschinger effect. It is therefore clear that these aspects of the material behavior must be accounted for accurately in order to obtain accurate springback predictions. However, one last and surprising aspect of material behavior that was observed with some draw-bend tests must be discussed: *time-dependent springback*.

Time-Dependent Springback The experimental results shown in Figure 42.2b were obtained for a tool radius-to-sheet thickness ratio (R/t) of 10. They showed little variation with forming rate or with time after the unloading step. However, process conditions which produce large springback angles (particularly for small R/t ratios) also produce significant time-dependent deformation for aluminum alloys after removal from the testing fixtures. Figure 42.5 shows the shapes for a single draw-bend specimen for times up to 7 months following testing.

Since first reporting these results (Wagoner et al. 1997), this phenomenon, which appears to have been previously neglected in the literature, has attracted interest in industry (Kuwabara et al.1996a, Tang 1996) and academia (Focellese et al. 1998). The impact of a part changing shape after forming has implications for replacement parts, which may meet specifications immediately after production, but then change during shipping and storage. Initial thoughts on the origin of time-dependent springback revolve around anelasticity (Zener 1948, Burton

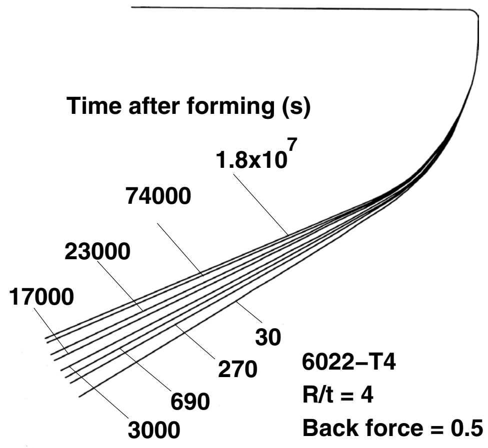

Figure 42.5: Measured time-dependent springback for a single draw-bend specimen for various times following deformation.

and Reynolds 1974, Ke 1994, Todd 1994, Wang et al. 1994), relaxation of the mechanical-threshold type (Beaudoin 2001), or room-temperature creep (Chandler 1987, Yang et al. 1988, Yang and Wang 1991, Sgobba et al. 1994, Sgobba et al. 1995) to relieve internal residual stress.

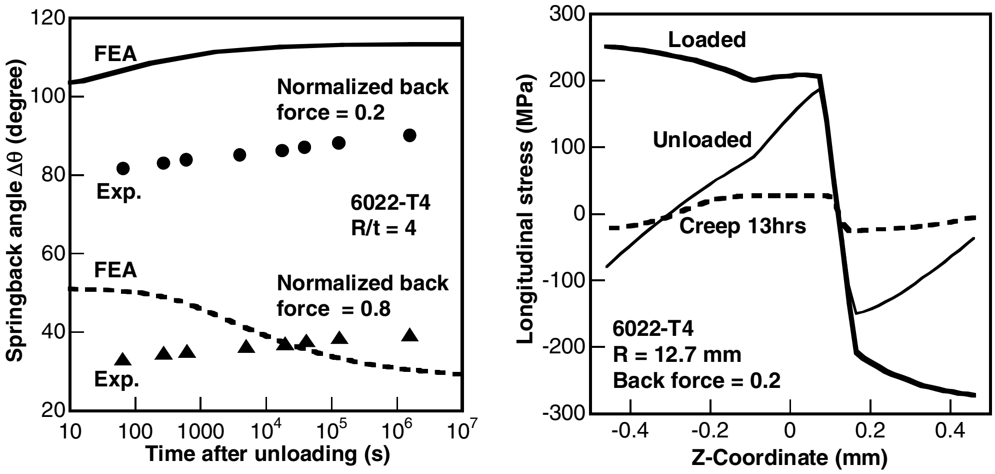

Figure 42.6: Preliminary time-dependent springback simulations based on standard creep data for pure aluminum: a) comparison of shape change with experiments, b) stress distribution through the thickness of the sheet.

Preliminary simulations based on a model of room-temperature creep to relieve internal stress at the end of the forming operation have shown reasonable qualitative agreement with the form of the time dependence of springback angle, Figure 42.6a. The corresponding re-arrangement and relaxation of internal stress is shown in Figure 42.6b. It should be emphasized that these results are very preliminary; they make use of very simple material laws with isotropic yield, and the creep law is for pure aluminum. Nonetheless, they demonstrate that such simulations can be carried out and that the form of the governing time dependence is unlikely to be simple.

42.5 Conclusions

Springback is an apparently simple phenomenon involving the elastic shape change of a body when deforming loads are removed. Nonetheless, the current inability to predict springback with accuracy costs manufacturers and consumers hundreds of millions of dollars per year in terms of lost production and remedial efforts. The complexities behind the simple façade of springback have been discussed.

The literature dealing with springback has been reviewed in terms of analytical methods, experiments, and numerical simulations. Much of the literature base is of limited use in predicting springback pertaining to industrial sheet forming operations. In many experiments the sheet tension has not been controlled reliably and many simulations apply to simpler situations than those encountered at the radius of a die cavity in common forming tooling.

Recent results by the author and coworkers for draw-bend springback has been summarized. Combined forming and springback simulations are shown to be much more sensitive and time-consuming than forming simulations alone, on the order of 100 times more CPU intensive. For even nominally 2D situations, 3D elements are required to take into account the anticlastic curvature that develops, and elastic-plastic unloading has been noted.

When numerical issues are addressed adequately, significant complexity of material behavior must also be considered to obtain accurate springback predictions. The material issues include the need for anisotropic yield functions that consistently reproduce both yield stress and strain ratio variation with direction, and novel hardening evolution formulations to reproduce the Bauschinger effect accurately. When these concerns are addressed, however, excellent springback predictions can be made.

The first reported observations of time-dependent springback of aluminum alloys were summarized and preliminary ideas of how this phenomenon can be modeled presented.

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