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Die design method for sheet springback

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Abstract

A new method for designing general sheet forming dies to produce a desired final part shape, taking springback into account, has been developed. The method is general in that it is not limited to operations having particular symmetry, die shapes, or magnitude of springback shape change. It is based on iteratively comparing a target part shape with a Finite Element-simulated part shape following forming and springback. The displacement vectors at each node are used to adjust the trial die design until the target part shape is achieved, hence the term "displacement adjustment method" (DA) has been applied. DA has been compared with the "springforward" method of Karafillis and Boyce (K&B), which is based on computing the constraint forces to maintain equilibrium following forming. DA was found to converge in cases when K&B does not, and in cases when both methods converge, DA is many times faster. In general, i.e. nonsymmetric parts, K&B can return inaccurate results whereas DA does not. The suitability and application of the two methods is discussed, along with the origins of the differences.

Keywords: Springback; Die design; Sheet metal forming; Contact pressure; FEM; Simulation

1. Introduction

Springback of sheet metal parts after forming causes deviation from the designed target shape and produces downstream quality problems and assembly difficulties. Its economic impact in

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terms of delayed production, tooling revision costs, and rejection of unqualified parts is estimated to exceed \$50 million per year in the US automotive industry alone [1].

Several approaches to controlling springback have been employed. Most of them focus on mechanical methods for increasing sheet tension during sheet bending [2–7], which dramatically reduces the magnitude of springback. This approach is preferred wherever possible because the small remaining springback is relatively insensitive to natural variations in process and material conditions, thus promoting a consistent response. However, this approach is unsuitable for many forming operations where increased sheet tension causes fracture of the sheet. For certain special operations, proper selection of R/t (radius of curvature to sheet thickness) or other parameters can be used to reduce springback [8–16].

An alternate approach to simply reducing springback lies in designing tooling in such a way to compensate for springback. That is, the springback may remain large while the final part shape would closely approximate that of the target part shape. The first step in implementing such a strategy is the accurate prediction of the springback phenomenon.

1.1. Springback prediction

Until recently, accurate springback prediction was only available for pure bending via empirical handbook rules or simple analysis, and for a few other specialized two-dimensional geometries [17–24]. Usually such results apply to very simple shapes with constant radii of curvature, and are based on well-studied materials such as mild steel.

There has been a growing interest during the past decade in using finite element methods for springback prediction following forming of arbitrary shapes. While apparently simple in concept, the prediction of springback has proven challenging for a variety of reasons, including numerical sensitivity, physical sensitivity, and poorly characterized material behavior under reverse loading and unloading conditions. However, it has also been shown that accurate springback prediction is achievable when these aspects are taken into account [25]. A full review of springback prediction is beyond the scope of this paper, but recent developments in these areas are cited more fully elsewhere [25–28].

1.2. Compensation for springback

Assuming that springback can be predicted accurately, there still remains the problem of how to use such results to arrive at a suitable die design to produce a target part shape. That is, the springback predictions allow "forward" analysis of forming and springback, while a "backward" analysis is needed to work from these results back toward an optimized die design. It is this second step of springback compensation that is addressed in the current work.

Simple bending operations involving constant radii of curvature can be designed to account for springback using handbook tables, which are usually available for a limited number of materials and sheet thickness. For alternate materials, varying radii of curvature (i.e. arbitrary curves), or arbitrary or three-dimensional shapes (with compound curvature), springback compensation has traditionally been carried out by simple trial-and-error, or by a guided form of this [29–31]. This usually takes place during the die tryout stage in a manufacturing plant, at great expense and time. The method is highly dependent upon the skill, experience, and luck of those carrying out the procedure. For complex aluminum panels, more than 6 months can be required during die tryout to correct springback error [32]. In principle, the trial-and-error method can be applied equally in a simulation framework, but this approach requires accurate springback prediction capabilities and can be as time consuming as experimental methods when a mechanism for guiding subsequent die design iterations has not been established.

It is clear that a method is needed for guiding die design to compensate for springback (backward direction) using sophisticated springback prediction capabilities (forward direction). Such a development was reported by Karafillis and Boyce [33–35]. This method, denoted by its authors as "springforward" (and denoted briefly in this paper as K&B) will be presented more fully later. It may be used with any finite-element (FE) program and is in principle a general method. However, as will be shown later, its application suffers from lack of convergence unless the forming operation is symmetric or has very limited geometric change during springback.

The only other approach to automated springback compensation found in the literature is based on an optimization strategy [36–38]. It involves a gradient method and a sensitivity analysis. This method involves considerable complexity in formulation and implementation as part of a special-purpose finite element program, and thus is not readily implemented with existing analytical tools.

The current paper presents an alternate closed-loop design method that avoids many of the limitations of K&B while maintaining its generality and ease of implementation. Designated the "displacement adjustment" (DA) method, it uses simulated forming and springback displacements in the punch travel direction to predict the next die design iteration. A similar approach has been utilized in practice via experimental iteration [39,40]. As will be shown through several arbitrary two-dimensional examples, DA offers several advantages, including excellent convergence rate, ease of implementation, and considerable generality.

2. Displacement adjustment method

The concept of the DA method is to move the surface nodes defining the die surface in the direction opposite to the springback error. In the first version presented here, such compensation is only made in the y direction (Fig. 1a), parallel to the punch travel direction. Furthermore, only two-dimensional forming operations from initially flat blanks are considered, although extension to more general cases should be straightforward. The procedures are depicted in Fig. 1a. First, a flat sheet of metal is deformed to a trial die shape (for the first cycle, the trial die shape is the target shape). After springback, the springback shape is compared with its target. The shape error is defined as $\Delta\vec{y}^i$, which is the vector of y coordinates of the target, less the y coordinates of the springback shape for the ith iteration. At Step 4, the $\Delta\vec{y}^i$ is added to the current die shape nodal positions, obtaining new tooling shape of $\vec{X}^{i+1}_{tool}$. For the next cycle, a flat sheet is deformed to this new tooling shape. If the springback shape is not within a specified tolerance of the target (checked at Step 3), another iteration will be conducted.

Since DA is a numerical method, its usefulness relies on the accuracy of springback prediction. With proper care taken in the simulation of springback, including material laws and contact conditions, accurate die shapes may be obtained. DA has recently been used to design a die shape to produce a contact heating device [41].

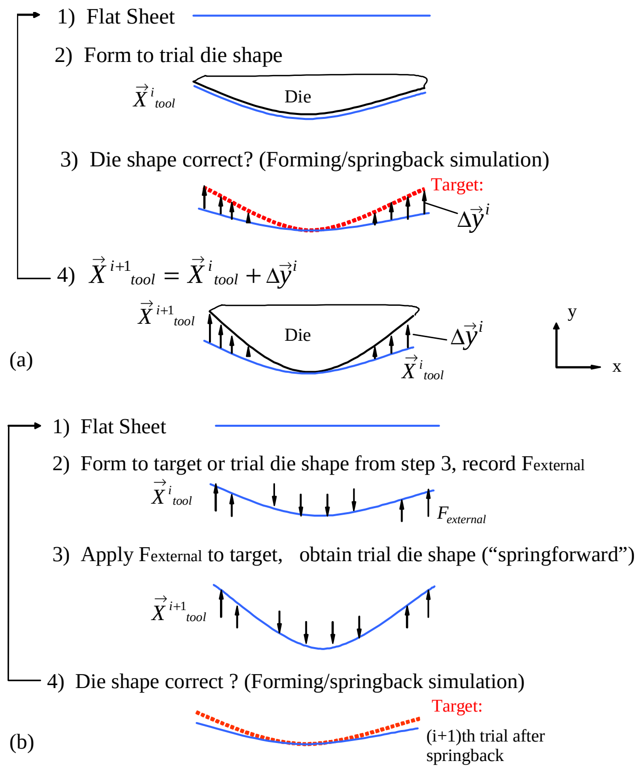

Fig. 1. (a) Flowchart of DA method; (b) flowchart of K&B method.

Fig. 1b contrasts K&B with DA. A flat sheet is first deformed into the target shape and the external forces are recorded. At Step 3, the target shape is elastically loaded by the recorded external forces and the next trial die shape is obtained (the same shape as the deformed blank shape at the end of this step). This is the “springforward” step. The accuracy of the trial die shape is next checked by doing a forming and springback simulation. If the resulting springback shape is not the same as the target, another cycle will be carried out from Step 1. Now a flat sheet is deformed to the trial die shape just obtained, instead of being deformed to the target shape as in the first cycle. External forces are recorded and applied to the target shape. A new trial die shape will be obtained. The new trial die shape will be checked again at Step 4, and iterations will continue until the target part shape is attained within a specified tolerance.

Karafillis et al. applied K&B successfully to two axisymmetric part shapes [34]. After 3 cycles, the shape errors could be reduced to 15% of their original values. However, in these two examples, no material draw-in was allowed and springback error was measured before the binder was removed (thus the deformation was stretch-dominated and the springback very limited). Later, the authors [35] extended the K&B method to 3-D geometries. Material draw-in was allowed in this work and the shape error was measured after removal from the tooling. The designed die shapes were capable of producing desired part shapes after several cycles. The results were verified by experiment and excellent agreement was obtained.

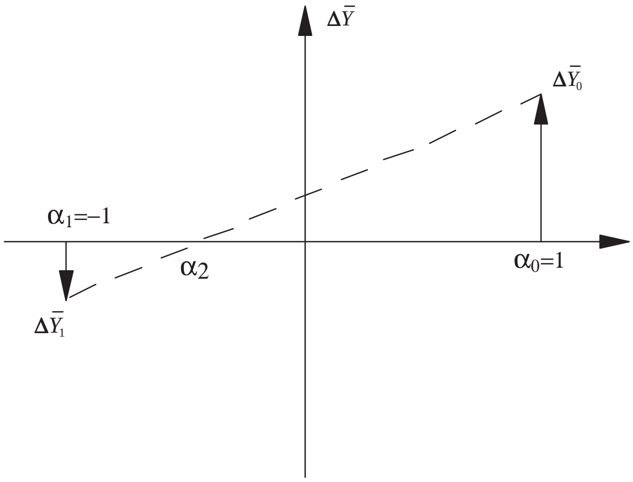

Fig. 2. Selection of α in a variation of K&B.

While K&B uses traction forces to do the "springforward" simulation, other variations have been proposed [35,42–44]. In these variations, the internal forces after forming are applied to the target shape but with the opposite sign. This produces a similar shape to applying external forces, but eliminates potential buckling instabilities. The internal force vector $F_{internal}$ can also be decomposed into an internal membrane force vector $F^{membrane}_{internal}$ and internal bending moment vector $F^{moment}_{internal}$. Since it was observed that the contribution of $F^{membrane}_{internal}$ is not important for springback phenomena [35], only $F^{moment}_{internal}$ need be applied during the springforward step. Unfortunately, the use of $F_{internal}$ in such an algorithm usually demands access to a FE program's coding, making it impractical for many commercial program users who do not have access to that code.

Another improvement involves applying only a fraction of $F_{internal}$, say $\alpha F_{internal}$, where α is a scalar multiplier [42–44]. If $\alpha = 1$, it simulates regular springback phenomena. If $\alpha = -1$, it is full springforward simulation. The value of α can be optimized during iteration by the simple interpolation shown in Fig. 2. $\Delta\bar{Y}_0$ is the initial shape error after springback, and $\Delta\bar{Y}_1$ is the error produced by the die shape after one cycle of springforward simulation. Value of α for the next iteration, i.e. α_2, is determined by interpolation as shown in Fig. 2. This method may improve convergence for large springback situations.

3. Results

In order to test the usefulness, convergence, and accuracy of DA, several two-dimensional and one three-dimensional part shapes were subjected to die design using DA and K&B.

3.1. Arc bending

An arc-bending problem (Fig. 3) was chosen as the first example because of its simplicity. ABAQUS STANDARD 5.8 [45] was used in FE simulations. The punch and die are treated as

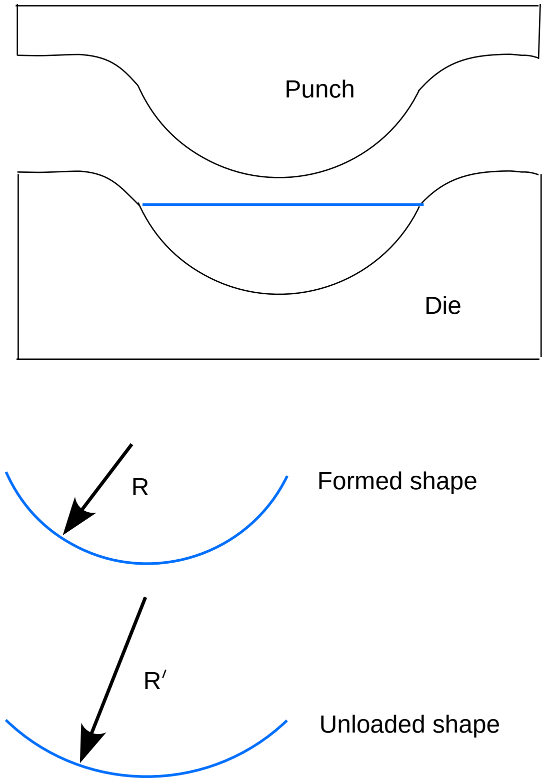

Fig. 3. Arc-bending test.

rigid bodies. There are 100 beam elements along the sheet length (Element B21). The beam section is 0.4572 mm in width and 0.1778 mm in thickness. There are 51 integration points through the thickness: the number determined to be necessary [46] to accurately model springback behavior. The material used in simulation has Young's Modulus = 207 GPa, Poisson's ratio = 0.3 and obeys the Holloman hardening equation:

$$\sigma(\text{MPa}) = 307 + 4460\varepsilon^{0.76}. \tag{1}$$

These properties correspond to an FeAl intermetallic alloy used for experiments reported later. Coulomb friction formulation was adopted with friction coefficient equal to 0.2.

As shown in Fig. 4, after just 1 cycle of DA iteration the springback error becomes negligible. The springback shape is nearly identical to the desired target (Fig. 4a). Fig. 4b shows the normalized error of DA iteration. It is defined as the root mean square shape error for the ith DA cycle over that of the initial cycle, where RMS shape error is calculated as

$$\sqrt{\frac{\sum_{k=1}^{N} \Delta y_k^2}{N}}, \tag{2}$$

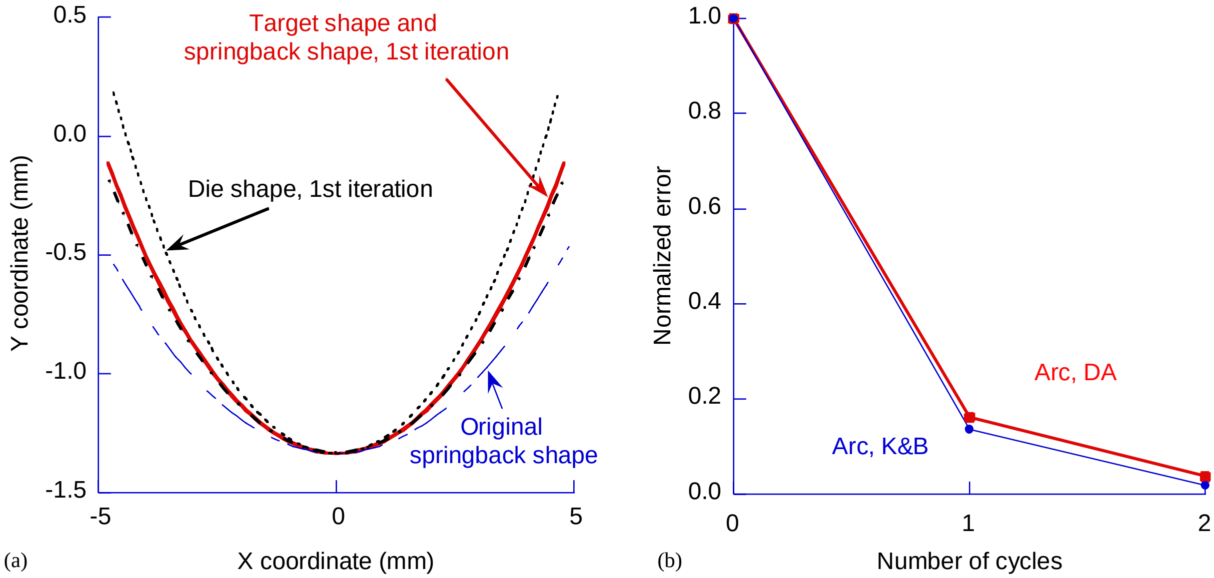

Fig. 4. Arc-bending results: (a) Die and part shapes, DA method; (b) convergence behavior of DA and K&B compared.

in which Δy_k is the shape difference in the y direction for the kth node and N is the number of nodes. K&B iteration was also carried out, with nearly identical final die shape and convergence rate (Fig. 4b).

3.2. U-channel bending

The successful implementation of DA on the arc problem encouraged attempts at more complicated cases, for example, U-channel bending. This problem was selected because of its wide interest in literature [37,47,48].

Fig. 5a shows the target U-channel shape along with results from the DA method. Again, since the sheet metal is not clamped during forming, this is a pure bending case. The same material properties as in the previous problem are used in simulation. It takes only 3 cycles of DA iteration to find an appropriate die shape to minimize springback error. When K&B is applied to the same problem, reduced springback error is achieved in up to approximately 20 cycles, after which the error oscillates and decreases no further. The die surface shapes produced by the two methods are similar, and the resulting springback shapes are nearly identical. Fig. 5b compares the convergence rate of the DA and K&B methods on the U-channel bending problem.

3.3. Compound curvature example

In order to evaluate the generality of the DA method, the die design for a 3D sheet-formed part with compound curvature was considered, Fig. 6a. The part is symmetric with respect to the X–Z and Y–Z planes, so only one quarter was modeled. The actual finite element mesh is refined 10

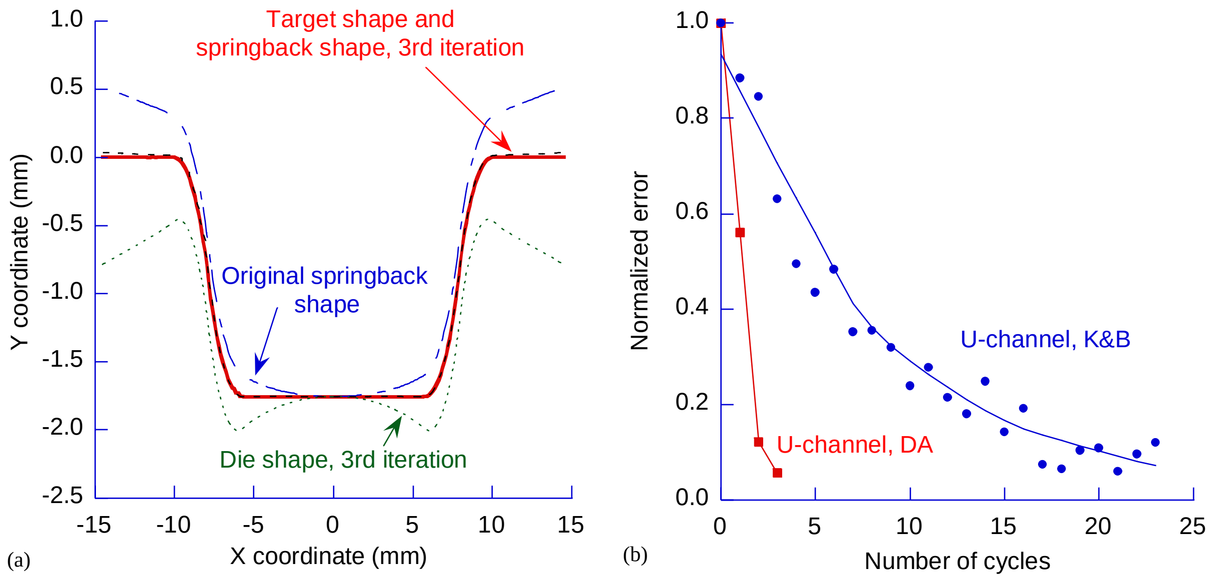

Fig. 5. U-channel bending results: (a) Die and part shapes, DA method; (b) convergence behavior of DA and K&B compared.

times in each direction from the mesh shown for illustration, i.e. 10,000 reduced-integration shell elements (ABAQUS type S4R) were used.

Die and part profiles along Y–Z and X–Z symmetry planes are shown in Figs. 6b and c. After one cycle of DA iteration, the springback profiles along the two symmetry planes approached their targets. The evolution of normalized springback error for the entire part is shown in Fig. 6d for both DA and K&B methods. The DA method minimized springback error rapidly, while the K&B method stopped improving after 4 iterations, and did not reduce error to an acceptable value.

3.4. Arbitrary 2D shape

The previous examples have symmetry axes or planes and simple analytic surfaces. A general part shape without symmetry and with no simple analytical representation, Fig. 7a, was chosen to test the algorithm more generally. For this problem, DA finds a suitable die shape in 5 cycles, while K&B showed no signs of convergence at up to 25 iterations (Fig. 7b). The best result by the K&B method for this problem is the springback shape achieved after 13 cycles (Fig. 7a), but significant shape error remains.

The limitations which prevent K&B from converging to an accurate solution will be addressed in the Discussion section of this paper.

3.5. Experimental verification

In order to verify the ability of DA to produce accurate die designs, an arbitrary target shape was chosen. DA was applied to find the appropriate die design, and this die design was

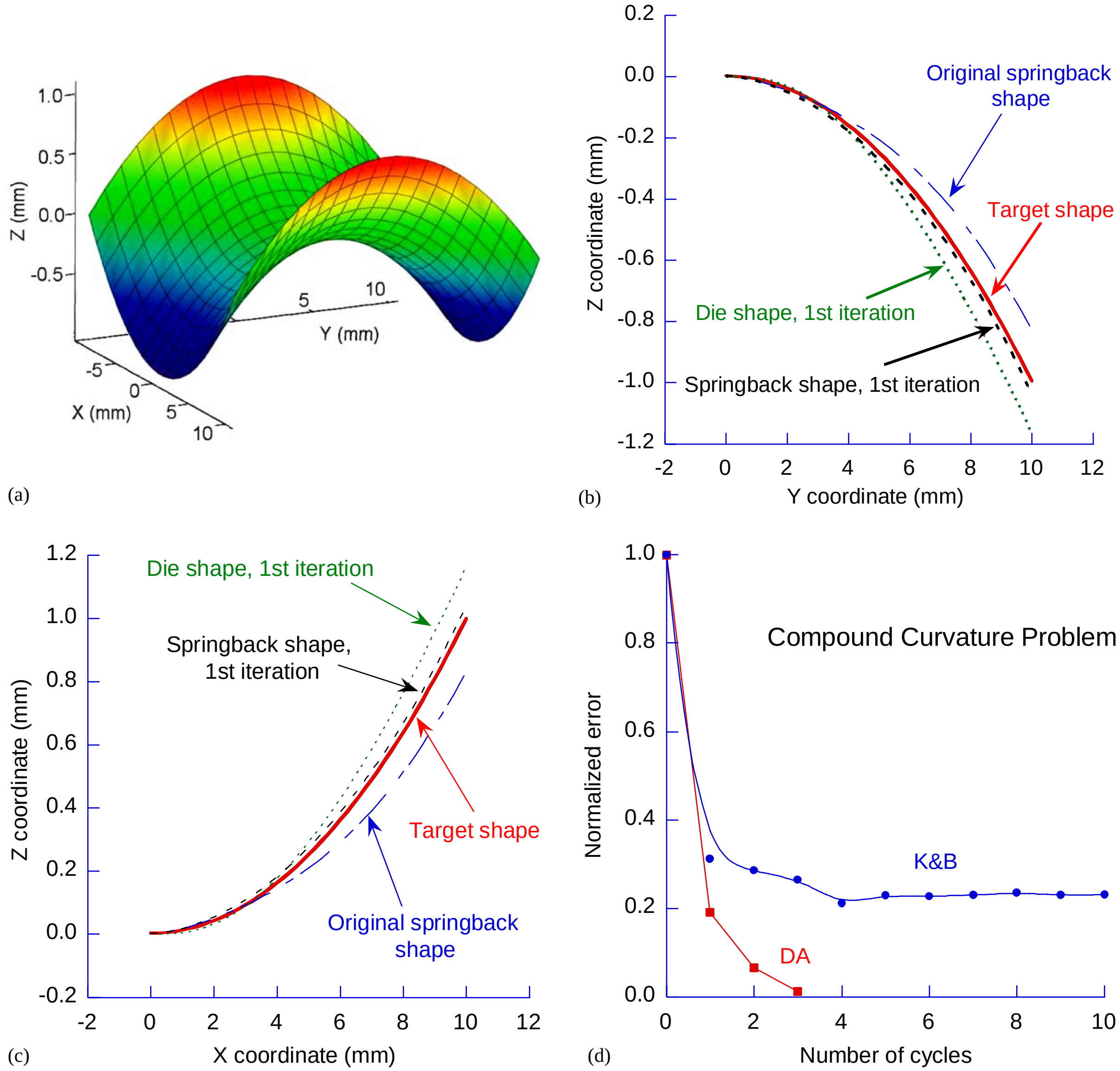

Fig. 6. Three-dimensional compound curvature bending results: (a) Target part shape; (b) die and part profiles along the Y–Z symmetry plane, DA method; (c) die and part profiles along the X–Z symmetry plane, DA method; (d) convergence behavior of DA and K&B compared.

used to machine tooling at Alabama Research and Development, ARD [49]. Parts were then formed from FeAl sheet at ARD using a punch of the predicted design and a polyurethane cushion to enforce conformity of the part to the punch profile during the

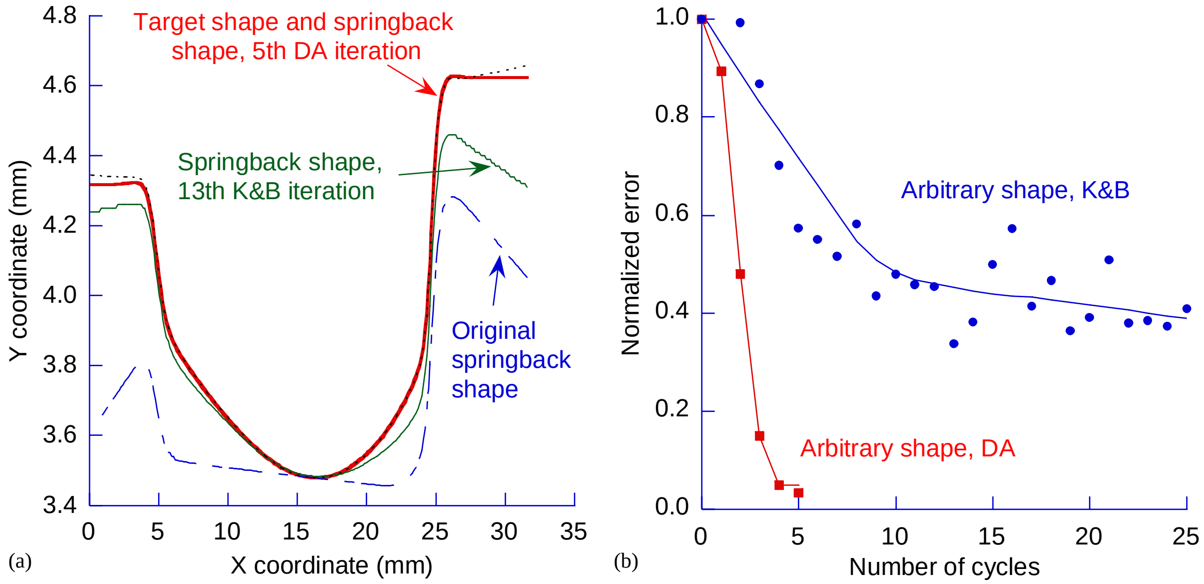

Fig. 7. Arbitrary 2D shape results: (a) Part shapes, DA method; (b) convergence behavior of DA and K&B compared.

forming operation. Following forming and springback, the part shapes were measured using a laser optical profiler. Details of these procedures and equipment have been presented elsewhere [41].

Two die sets were constructed with the target part shape in mind, one labeled "ARD" and one labeled "DA". For the ARD set, a punch shape was designed at ARD using circular arc segments and straight line segments based on known springback correction curves. A punch was made, a part formed, and the shape compared with the target. On a trial and error basis, another die was then machined and again the shape compared. The third set of tooling created by this procedure was judged to be satisfactory.

In contrast, the arbitrary DA tooling (not circular arc segments and line segments) was designed according to the procedure outlined in Fig. 1a and the final tooling machined according to these specifications. Five DA iterations were required with a total computation time of approximately 2.5 h using a Pentium 4 personal computer (2.8 GHz).

Fig. 8 summarizes the results of this exercise, with measured shapes averaged from 8 repeated forming operations. The ARD die design produces parts with a variance from the target part shape approximately 5 times greater than the DA method, in spite of the three iterations using physical die sets. Some of the variance occurs because the springback correction curves are approximate. The magnitude of this effect may be seen by comparing the measured part shape with the predicted springback shape for the ARD design. This simulation was performed using the same forward analysis as in the DA case. The remainder of the variance occurs by limiting the part and die design to arcs and lines, whereas the actual target part shape involves smoothly varying radii of curvature.

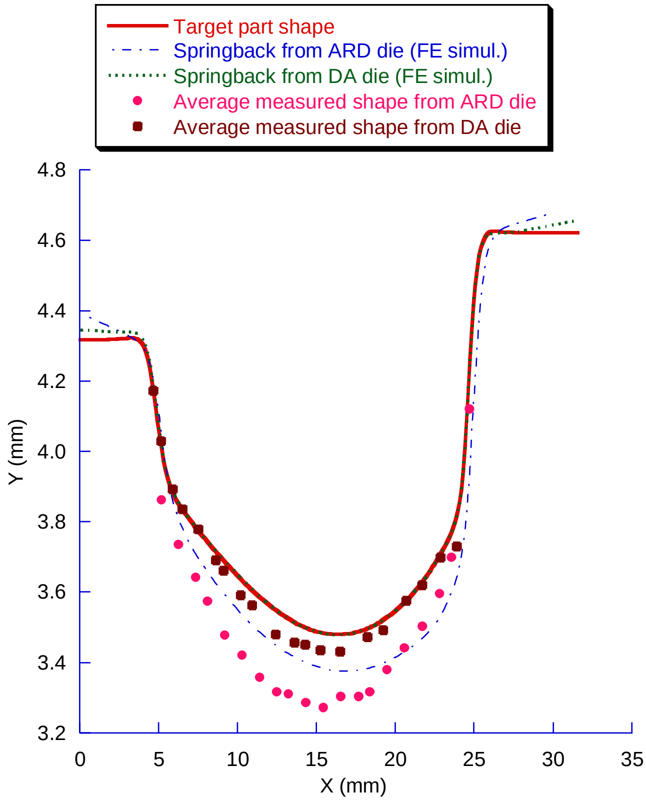

Fig. 8. Experimental validation of the DA method.

4. Discussion

The limited results presented in the previous section raise several questions about the relative convergence and accuracy of the K&B and DA methods, and the generality of each approach. In this section, we develop two small models to help reveal the convergence behavior and the effect of material properties and magnitude of springback on this convergence.

4.1. Application to arbitrary shapes

As shown in Figs. 5b, 6d and 7b, K&B converges very slowly or not at all under some conditions. The question is what causes such behavior? Upon examination of the iterative results, it appeared that the performance of K&B depends intimately on the choice of boundary conditions. That is, some degrees of freedom in any FE simulation must be constrained to avoid rigid body translation and rotation. The choice of such conditions and where they are applied is not unique, but they play an important role in K&B.

In order to examine the effect of constraint choices on K&B, the springback problem of a symmetric shape was studied, as shown in Fig. 9a. The target part shape is constructed by

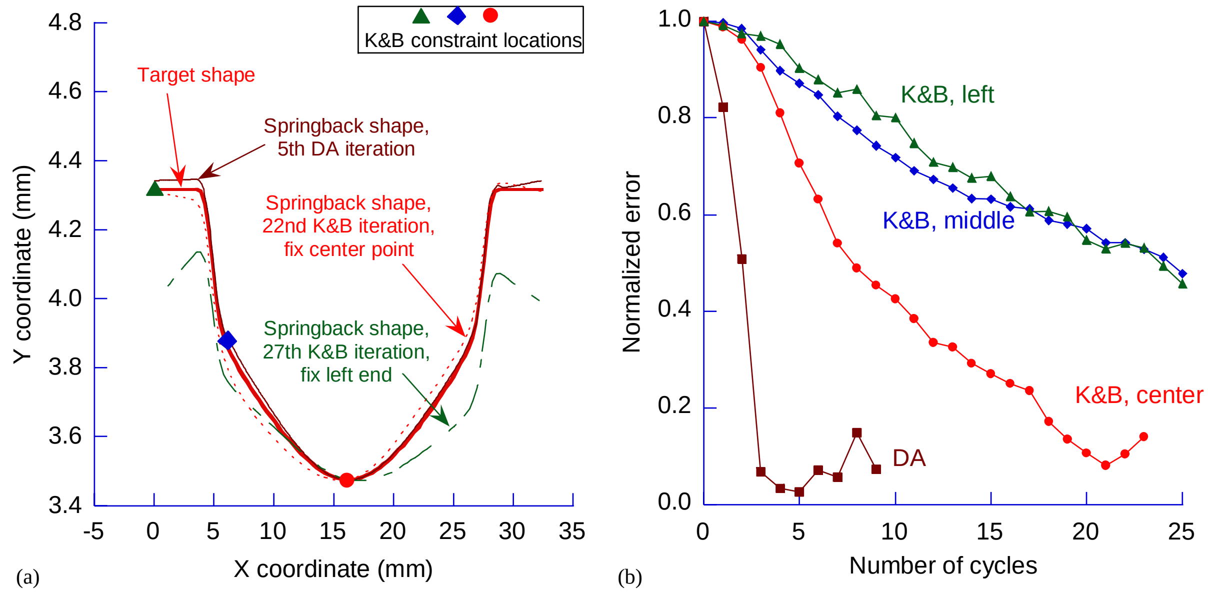

Fig. 9. Symmetric arbitrary 2D shape results: (a) Part shapes; (b) convergence behavior of DA and K&B compared.

projecting the left half of the general shape in Fig. 7a (from left end to the lowest point which has zero slope) to the right side, thus making a symmetric part. During the "springforward" step, the traction forces were applied to the target shape with a point fixed in displacement and rotation to avoid rigid body movement. Depending upon which point is fixed, 3 cases were studied—constraining the leftmost point, the central point or a point between the previous two. The positions of these points are indicated in Fig. 9a.

As shown in Fig. 9b, the choice of constraint has a large effect on not only the convergence rate but possibly on the converged solution from K&B. For the constraint at the symmetry point, K&B eventually approaches a low-error solution, whereas for other constraints there is no indication that a low-error solution will be found. In all cases, the convergence rate is very slow, on the order of 20 iterations for this example compared with 5 iterations for DA (with residual error less than 3% of the initial error).

There does not appear to be any obvious way to modify K&B to overcome the problem of constraint choice for general forming operations. (For symmetric parts, choosing the symmetry point for constraint works well.) Unlike the part considered in Fig. 9, there is in general no symmetry point for a forming operation. When the forces are applied in the K&B "springforward" step, these forces interact with the constraints to either deny convergence or to produce apparent convergence, but of the wrong die shape. It is apparently for this reason that only symmetric die shapes were studied by Karafillis and Boyce [33–35], who did not report such problems with the method.

In contrast, DA does not rely on evaluation of forces outside of the FE loop and thus the question of constraint does not appear. Fig. 9b shows that for this symmetric example DA reduces the variance to less than 3% of its initial value after just 5 iterations.

4.2. Sensitivity of DA and K&B methods

In order to investigate the sensitivity of DA and K&B to variations in process parameters, a very simple two-element model was constructed, as shown in Fig. 10. A beam 10 mm in length, 1 mm width and 0.1 mm thickness was modeled using ABAQUS B21 elements with 51 integration points through the thickness. The left node is constrained against translation and rotation and the center element is constrained vertically. A y-displacement is applied to the right node and the node is then released. The error for this model is defined as the difference between the y coordinate of the springback position and the target position, and a tolerance of 0.01 mm was chosen arbitrarily.

Material behavior was first studied for a target node end displacement of 1 mm. The elastic modulus was varied with a non-hardening plastic law with yield stress of 100 MPa. The following chart reports the number of iterations to achieve the required tolerance.

	DA	K&B
$E = 140\,\mathrm{MPa}$	1	1
$E = 70\,\mathrm{MPa}$	2	2
$E = 35\,\mathrm{MPa}$	4	6

The results show little difference between the two methods, although K&B appears to be more sensitive to the magnitude of springback. A similar test performed to consider the role of material hardening showed no differences in the two techniques.

For the final sensitivity test, the severity of the forming operation was modeled by displacing the right node by a distance of either 1, 2, or 2.5 mm. For this test, $E = 35\,\mathrm{MPa}$ and the yield stress was 100MPa, with no strain hardening. The results, depicted in Fig. 11, show that DA convergence is somewhat faster than K&B, but DA allows oscillations in both directions from the target shape during iteration. This may in some circumstances cause convergence problems for large springback cases. This problem has not been encountered thus far, but a fractional application of error displacements (similar to that applied by Wu et al. [42–44] for K&B forces, as discussed earlier) should provide for a slower, more uniform convergence behavior if required.

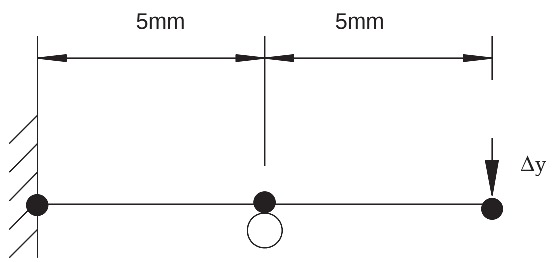

Fig. 10. Model setup for the sensitivity analysis.

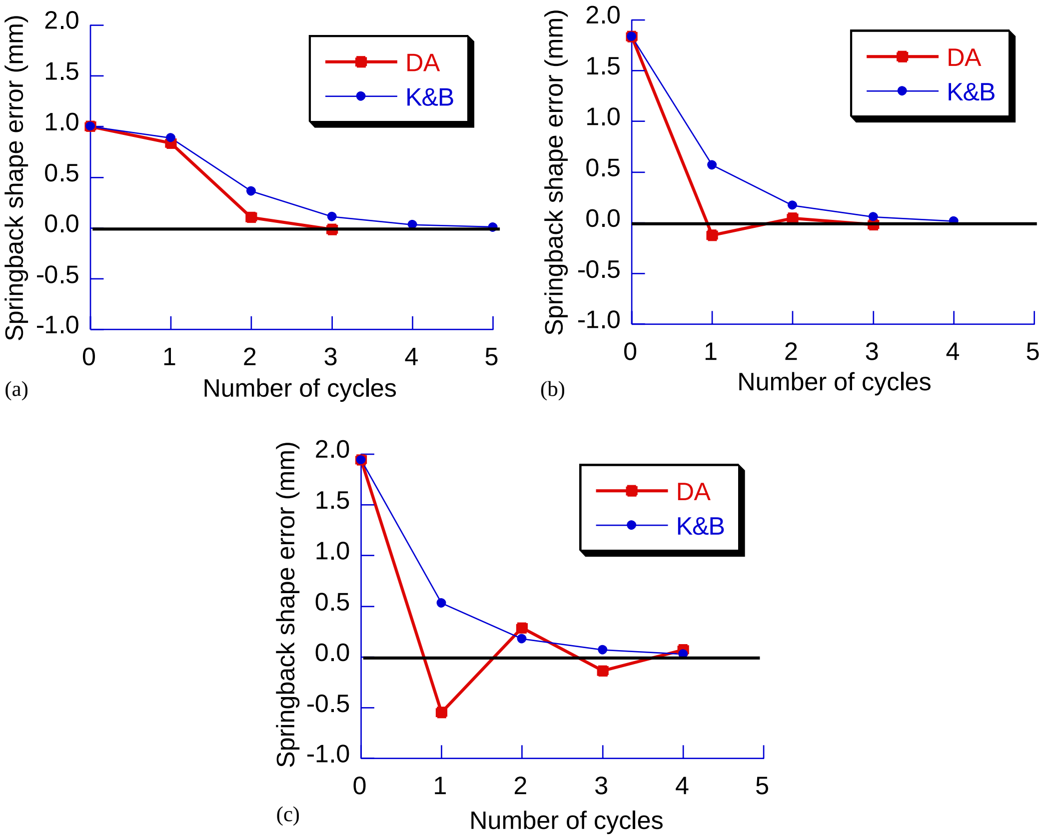

Fig. 11. Dependence of K&B and DA on deformation geometry. The target shape is the shape with Δy equal to (a) 1 mm, (b) 2 mm, (c) 2.5 mm.

5. Conclusions

A method for the design of sheet metal forming dies to produce a specified part shape, taking into account springback, has been developed. The "displacement adjustment" method (DA) is an iterative technique based on comparing a target part shape with a formed-and-unloaded part shape simulated using finite elements. In contrast, the "springforward" method (K&B) in the literature makes use of FE-simulated forces for a similar iterative procedure. DA has been presented and tested for several challenging cases, in both 2D and 3D, and compared with K&B results. The tests and analyses reveal the following conclusions:

- DA converges rapidly, usually within five iterations. In contrast, K&B converges more slowly for some cases or may converge to incorrect die shapes for other cases. However, DA iterations may oscillate while K&B iterations show steadily decreasing error.
- DA converges for cases involving very large springback shape changes, of the order of the forming shape change.

- DA does not rely on part symmetry for convergence. K&B often fails to obtain the correct solution for non-symmetric parts.
- The K&B convergence limitations originate from the need to evaluate external forces, which depend upon the choice of boundary conditions. While boundary conditions may be readily chosen for symmetric forming operations, there is no obvious method for choosing the proper boundary conditions for non-symmetric forming operations.
- DA has been confirmed by experiment to produce accurate die shapes and final part shapes with no trial-and-error and with few iterations. In contrast, dies constructed using traditional methods had significant errors of final part shape.

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